CBDC and business cycle dynamics in a New Monetarist New Keynesian model*

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Abstract

In this paper, we study how the existence of a central bank digital currency (CBDC) affects the response of the economy to business cycle and financial shocks as well as shocks to the means-of-exchange function of money. To this end, we integrate a New Monetarist-type decentralised market that provides a role for bank deposits and CBDC as means of exchange into a New Keynesian model with financial frictions. We find that the existence of CBDC allows the central bank to separately target the store-of-value and the means-of-exchange function of money and thereby opens up a new channel to stabilise the economy. This channel operates through the liquidity premium and implies a trade-off between payments efficiency, bank funding conditions and the opportunity cost of holding money.

Keywords: Central bank digital currency, monetary policy, DSGE, search and matching.

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1 Introduction

In the past few years, an increasing number of central banks have started to consider the potential provision of central bank digital currency (CBDC) to the wider public for payment purposes (Boar and Wehrli, 2021; Boar et al., 2020). Such a provision would raise several policy questions, in particular for monetary policy transmission and financial stability, especially if CBDC were to replace bank deposits on a larger scale. A substitution of bank deposits for CBDC could raise banks' funding costs, with potential consequences for bank lending rates and credit provision through banks to the economy, which could change monetary policy transmission and might cause financial instability (Group of Central Banks, 2021). Policy makers are therefore mindful of "doing no harm" to public policy objectives when considering a potential issuance of CBDC (Group of Central Banks, 2020).

In this paper, we investigate the consequences that the potential existence of a CBDC could have on monetary policy transmission and the responses of output and inflation to macroeconomic shocks. To this end, we integrate a decentralised market in the spirit of Lagos and Wright (2005) into a New Keynesian DSGE model with financial frictions that is based on Gertler and Karadi (2011) by following the approach of Aruoba and Schorfheide (2011). This setup contains a centralised market with a New Keynesian structure, followed by a decentralised market, in which CBDC and deposits are perfect substitutes and essential to enable consumption. By combining the New Monetarist setup of decentralised transactions with the New Keynesian business cycle dynamics, this approach is well suited to analyse the properties of deposits and CBDC as means of payment with an explicit micro-founded role for money in the decentralised market. At the same time, we can investigate how business cycle fluctuations, financial frictions and monetary policy interact with the payment functions of different forms of money. To our knowledge, this approach has not been applied to the analysis of CBDC to date.

In modelling the micro-foundations of CBDC as a means of payment explicitly, we depart from DSGE models that include money in the utility function as pioneered by Sidrauski (1967).[1] As highlighted in central bank communication (see, e.g. Panetta (2021) for the digital euro), the payment function is central to the design of a CBDC. It is therefore important to reflect it adequately in model-based analyses. To study the transmission channels of CBDC and its implications for model dynamics, a structure that explicitly accounts for the use of CBDC as medium of exchange seems more appropriate than integrating CBDC indirectly through a money-in-the-utility-function (MIU) specification. While MIU is a convenient shortcut to modelling the relevance of money as a means of exchange, the use of such a shortcut risks neglecting relevant dynamics that, for instance, operate through the changes in the liquidity premium that arises from the value of money

[1] See, for instance, Niepelt (2020) or Gross and Schiller (2021).

as a means of exchange.[2]

Our results suggest that the existence of CBDC mitigates macroeconomic fluctuations in response to business cycle and financial shocks. This is particularly the case for shocks to capital quality or banks' net worth that transmit through banks' balance sheets in the centralised market. CBDC alleviates the financial friction that is generated by the banks' leverage constraint, which contributes to dampening the inflation response and reducing the persistence of the output response. Although standard monetary policy retains its ability to affect output and inflation effectively in the presence of CBDC, the existence of CBDC provides the central bank with a second policy instrument that allows the central bank to stabilise the liquidity premium. In that way, the central bank can separately target the store-of-value and the means-of-exchange function of money and thereby stabilise the economy through an additional channel that exploits a trade-off between payments efficiency, bank funding conditions and the opportunity cost of holding money.

This paper contributes to a growing literature on CBDC.[3] Questions related to financial stability have been investigated quite extensively, see for instance Andolfatto (2021), Williamson (2021), Keister and Sanches (2019), Böser and Gersbach (2020), Bitter (2020) and Chiu et al. (2019). The implications of different CBDC design parameters on equilibrium capital allocation, financial intermediation and welfare are studied by Assenmacher et al. (2021). These papers mostly employ a Lagos and Wright (2005) framework and compare how the existence of CBDC changes equilibrium allocations. However, they do not analyse how the dynamic responses of macroeconomic variables to business cycle or financial shocks are changed in the presence of a CBDC. Another strand of the literature focuses on possible reactions of the central bank to changes in bank funding conditions initiated by the existence of a CBDC. Brunnermeier and Niepelt (2019) and Niepelt (2020) argue that the introduction of CBDC would be neutral with respect to bank funding as the central bank could always undo the resulting effects on the banking sector, although limits to this neutrality result are also recognised (Fernández-Villaverde et al., 2021). Schilling et al. (2020) maintain that CBDC causes a trilemma for the central bank in the sense that it cannot ensure an efficient asset allocation, price stability and financial stability a the same time.

Issues related to monetary policy transmission and implementation are less well researched. Barrdear and Kumhof (2021) investigate consequences of issuing CBDC in a DSGE model and argue that a countercyclical CBDC policy rule could substantially improve the central bank's ability to stabilise the business cycle. However, their framework

[2]The Lagos and Wright (2005) model features a liquidity premium that directly reflects the transaction services CBDC is providing.

[3]Carapella and Flemming (2020), Aragão (2021) and Auer et al. (2021) survey the rapidly growing literature on CBDC.

does not capture the specific function of CBDC as a means of exchange. Our setup is close to Gross and Schiller (2021), who analyse the impact of a CBDC in a Gertler and Karadi (2011) type model with money in the utility function but do not account explicitly for the means-of-exchange function of CBDC.

The paper is structured as follows. Section 2 describes the model environment with the centralised and the decentralised market. In Section 3 we simulate the model and discuss the results. Finally, Section 4 concludes.

2 Model environment

The model consists of a centralised market (CM) and a decentralised market (DM). The centralised market is modelled as a New Keynesian model with financial frictions in the spirit of Gertler and Karadi (2011), while the decentralised market is modelled as a monetary search model providing microfoundations for the role of CBDC and deposits as a medium of exchange. Although the CM follows Gertler and Karadi (2011) closely, we introduced a few changes in order to link it to the decentralised Lagos and Wright (2005) market. One important change is that we need linear disutility of labour to ensure tractability of the model. With linear disutility of labour, agents in the centralised market choose their production and consumption plans in a way that offsets any changes in the allocation that result from their ability to trade on the decentralised market.[4] The two markets take place sequentially in each period, i.e. a period starts with the CM, after which the DM opens. Discounting takes place after the DM but not between the CM and the DM. Figure 1 depicts an overview of the model structure which includes five types of agents: households, financial intermediaries (banks), capital goods producers, intermediate goods producers and retail goods producers. As in Gertler and Karadi (2011), banks face an agency problem that introduces a leverage constraint and can lead to socially insufficient provision of credit and deposits. The central bank issues CBDC that serves as a medium of exchange alongside deposits. In this setup, the central bank can influence the economy through conventional policy measures and through the availability and desirability of CBDC.

Whereas money in the CM is modeled as a store of value, the DM provides a role for money as a medium of exchange. Households receive a preference shock at the beginning of the DM that determines whether they become sellers or buyers, thus creating a double-coincidence-of-wants problem. In the DM, households are anonymous and can either pay with bank deposits or with CBDC. Like Keister and Sanches (2019), we assume that sellers are endowed with a technology to recognise specific means of payment, i.e. CBDC or bank deposits, but that they cannot accept securities or other types of claims. In

[4]While this setup is a typical feature of Lagos and Wright (2005)-type models, it requires a different parameter calibration than Gertler and Karadi (2011) that, however, has only a limited impact on the form of the impulse responses to typical macro shocks.

practice, this would mean that sellers have the technology, such as card readers, installed to accept CBDC or debit cards. This setup makes money essential and allows us to study the interactions between the central bank's balance sheet, monetary policy and the payments function of money.

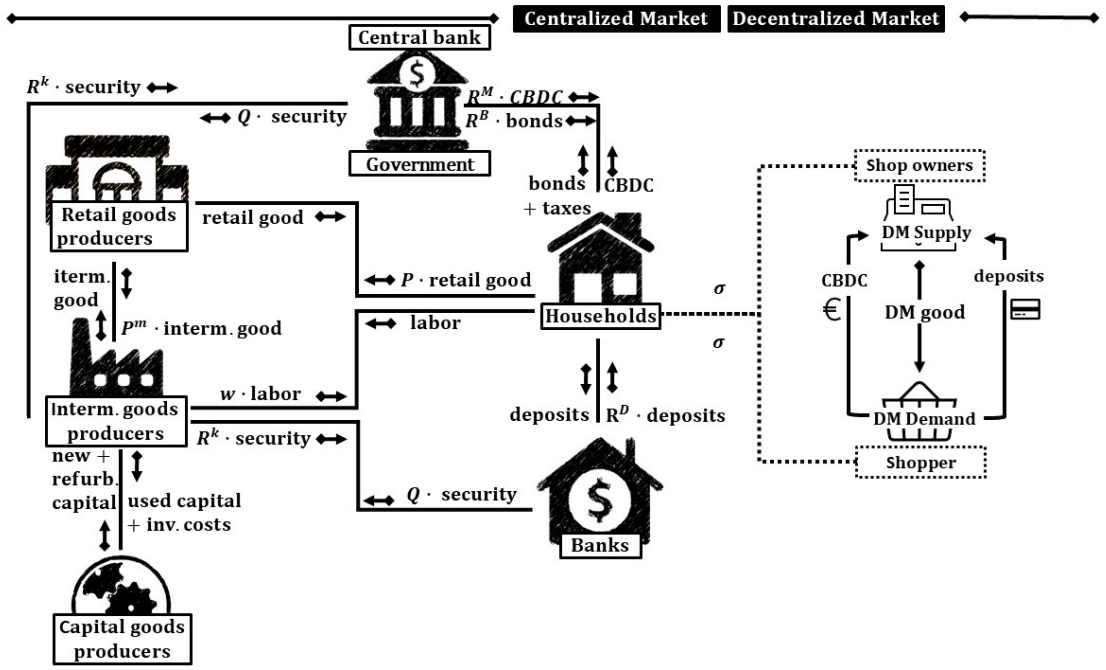

Figure 1: Overview of the model structure

We first describe the decision problem of households, banks and different types of firms in the CM before we turn to the framework and the decision problems of buyers and sellers in the DM.

2.1 The centralised market

The CM features the usual New-Keynesian frictions that allow monetary policy to have real effects. Retail goods producers operate under monopolistic competition and repackage the output produced by intermediate goods producers. Moreover, the model assumes sticky prices such that a firm can freely adjust its price with probability $1 - \gamma$ in a specific period whereas in all other periods it can only index its price to lagged inflation. In addition to these two frictions, the model includes investment adjustment costs for intermediate goods producing firms and an incentive constraint for bankers that creates a friction in the bank intermediation process. This financial friction on one hand magnifies the impact of shocks to banks' net worth on the real economy and on credit flows, while on the other hand, it also amplifies the effect of such shocks on the availability of deposits and thus affects the amount of transactions in the DM. We will discuss the financial sector

and how this friction plays out further below.

Households

In the CM, there is a continuum of identical households of measure unity. Each household has two different types of members: a fraction f of the household are bankers and a fraction $1-f$ are workers. Bankers remain bankers in the next period with a time-independent probability θ. This implies that in each period $(1-\theta)f$ bankers become workers and vice versa.[5] In the CM, households maximise their utility by choosing consumption C_t^{CM}, labour L_t as well as real CBDC holdings M_t, real deposits D_t, and real bond holdings B_t, taking the expected continuation value in the DM V_t^{DM} into account. The households' value function in the CM therefore is

$$V_t^{CM} = \max_{C_t^{CM}, L_t, M_t, D_t, B_t} \left\{ U(C_t^{CM}) - \chi L_t + E_t(V_t^{DM}) \right\} \tag{1}$$

with[6] $U(C_t^{CM}) = \ln(C_t^{CM})$ s.t. the budget constraint

$$C_t^{CM} + M_t + D_t + B_t = w_t L_t + T_t + R_t^M \hat{M}_{t-1} + R_t^D \hat{D}_{t-1} + R_t^B B_{t-1} + \Omega_t, \tag{2}$$

where w_t is the real wage, T_t denotes lump-sum taxes, R_t^M, R_t^D and R_t^B are the gross real rates of return on CBDC, deposits and bonds, respectively, and Ω_t is the net payout from the ownership of financial and non-financial firms, including the net cash-flow from trading state-contingent securities.[7] While CBDC, deposits and bonds are all financial assets, only deposits and CBDC can be called money as they are able to carry out transaction services in the DM. $\hat{M}_t$ and $\hat{D}_t$ are CBDC and deposit holdings that are carried over from the previous DM and depend on whether the household is a buyer, a seller or inactive in the DM, namely

$$\hat{M}_t = \begin{cases} M_t - m_t^B & \text{for buyers} \\ M_t + m_t^S & \text{for sellers} \\ M_t & \text{otherwise} \end{cases} \quad \text{and} \quad \hat{D}_t = \begin{cases} D_t - d_t^B & \text{for buyers} \\ D_t + d_t^S & \text{for sellers} \\ D_t & \text{otherwise} \end{cases}$$

where m_t^B and d_t^B are the amounts of CBDC and deposits spent in the DM and m_t^S and d_t^S are CBDC and deposits received in the DM (see Section 2.2). On aggregate, sellers get what buyers pay i.e. $m_t = m_t^B = m_t^S$ and $d_t = d_t^B = d_t^S$. As the households' utility function is log-linear, CBDC and deposit holdings will be identical for all households at

[5] This assumption ensures that banks face a leverage constraint.

[6] Similar to Aruoba and Schorfheide (2011), we abstain from modelling habit persistence which also simplifies calculations in the DM.

[7] Note that the budget constraint is expressed in real terms, i.e. in units of the CM consumption good. Consequently, M_t and D_t denote real CBDC and real deposit holdings. We discuss the evolution of the price level and inflation later on.

the end of the CM, irrespective of their role in the DM (Lagos and Wright, 2005), which ensures that the optimality conditions do not depend on individual state variables.

Optimality conditions for the CM (equations 3 and 4) require that the marginal utility of consumption in the CM, captured by ϱ_t^{CM}, equals the marginal disutility of labour relative to the real wage χ/w_t. The marginal value of assets in the CM, denoted by V_{t,M_t}^{CM}, V_{t,D_t}^{CM} and V_{t,B_t}^{CM}, consist of the costs of acquisition and its continuation value captured by V_{t,M_t}^{DM}, V_{t,D_t}^{DM}, V_{t,B_t}^{DM}.

$$V_{t,C_t^{CM}}^{CM} = \frac{1}{C_t^{CM}} - \frac{\chi}{w_t} = 0 \quad \Rightarrow \quad \varrho_t^{CM} \equiv \frac{1}{C_t^{CM}} = \frac{\chi}{w_t} \tag{3}$$

$$V_{t,a_t}^{CM} = -\frac{\chi}{w_t} + V_{t,a_t}^{DM} = 0 \quad \text{for} \quad a = \{M_t, D_t, B_t\} \tag{4}$$

The envelope conditions (equation 5) imply that the marginal value of assets in the CM consists of its interest payments, weighted by the marginal utility of consumption on the CM. This reflects that the value of assets in the CM originates only from its function as store of value. Contrary, in the DM money derives value only through its function as a medium of exchange. In this way, the modelling of the different functions of money is split into two distinct markets. This is handy for analytical purpose, however one should note that there is also an implicit transaction motive in the CM.

$$V_{t,\hat{M}_t}^{CM} = \varrho_{t+1}^{CM} R_{t+1}^{M}, \quad V_{t,\hat{D}_t}^{CM} = \varrho_{t+1}^{CM} R_{t+1}^{D}, \quad V_{t,B_t}^{CM} = \varrho_{t+1}^{CM} R_{t+1}^{B} \tag{5}$$

Banks

Banks are only active in the CM. They invest into shares of intermediate goods firm which are funded by household deposits $D_{j,t}$ and the bank's equity $N_{j,t}$. The balance sheet of a bank j can be written as

$$Q_t S_{j,t}^{B} = N_{j,t} + D_{j,t} \tag{6}$$

where $S_{j,t}^{B}$ is the quantity of financial claims of the intermediate goods producer that the bank holds and Q_t is their relative price. Bank deposits are used as a means of payment in the DM. Shocks to the banks' balance sheet thus will not only affect capital investment and production in the CM but also transactions in the DM.

Bankers maximise the expected discounted terminal net worth of their bank $V_{j,t}$

$$\begin{aligned} V_{j,t} &= \max E_t \sum_{i=0}^{\infty} (1-\theta)\theta^i \beta^{i+1} \Lambda_{t,t+1+i} N_{j,t+q+i} \\ &= \max E_t \sum_{i=0}^{\infty} (1-\theta)\theta^i \beta^{i+1} \Lambda_{t,t+1+i} \left[(R^S_{t+1+i} - R^D_{t+1+i}) Q_{t+i} S^B_{j,t+i} + R^D_{t+1+i} N_{j,t+i} \right] \end{aligned} \tag{7}$$

with θ being the probability that the bank continues its operations in the next period, $\beta\Lambda_{t,t+1+i}$ the stochastic discount factor that the bankers in period t apply to their earnings in $t+i$ and R^S_{t+1+i} the gross real return on the banks' investment in the intermediate goods producer. $(R^S_{t+1+i} - R^D_{t+1+i})$ is thus the return on the bank's investment into the intermediate goods firm, $R^S_{t+1+i} Q_t$, less the cost of funding this investment, $R^D_{t+1+i} D_{j,t+i}$, making use of the balance sheet identity in (6).

Banks are subject to an incentive constraint that limits the amount of deposits the households are willing to entrust them with and therefore the amount of investment. Specifically, it is assumed that households can only recover a share $1-\lambda$ of the banker's assets in case of a bankruptcy, see Gertler and Karadi (2011). To avoid that the banker prefers to divert a share λ from the funds invested in the project, the net worth of the bank, $V_{j,t}$, has to exceed the gain the banker can realise from such illicit diversion.

$$V_{j,t} \geq \lambda Q_t S^B_{j,t} \tag{8}$$

When the incentive constraint binds, banks' net worth constrains the amount of investment and deposits, implying that a shock to the banks' net worth will spill over into investment and production. As long as the expected discounted marginal gain from acquiring an additional capital claim is smaller than the diversion fraction, the incentive constraint will be binding[8]. As in Gertler and Karadi (2011), the constraint will always bind locally within the steady state under the adopted parametrisation.

Firms

The production sector follows Gertler and Karadi (2011) with a set-up that is standard in New Keynesian models. It consists of capital goods producing firms, intermediate goods firms and retail goods firms.

Capital goods producing firms operate in a competitive environment. They buy capital K_t from the intermediate goods firms at the end of period t, repair depreciated capital and build new capital, and then resell it to the intermediate goods firms. As the intermediate goods firm faces adjustment cost on net investments, capital goods producers may earn profits outside of the steady state, which they distribute in lump-sum form to

[8]For the derivation see the Appendix A.2.

their owners, the households. The discounted profit for capital goods producers is given by:

$$\max E_t \sum_{i=0}^{\infty} \beta^i \Lambda_{t,t+i} \left[Q_{t+i} I_{t+i}^N - \frac{\iota}{2} \left(\frac{I_{t+i}^N + I^{SS}}{I_{t+i-1}^N + I^{SS}} - 1 \right)^2 \left(I_{t+i}^N + I^{SS} \right) \right] \tag{9}$$

in which $I_t^N \equiv I_t - \delta(U_t)\xi_t K_t$ is the net capital produced, I_t is the gross capital, $\delta(U_t)$ is the depreciation rate[9], U_t is capital utilisation, ξ_t is a capital quality shock and I^{SS} is the level of investment in steady state. Following the literature, we assume quadratic flow adjustment costs on net investment (Christiano et al., 2005; Gertler and Karadi, 2011), with ι denoting a scaling parameter.

The intermediate goods firms, indexed by M, acquire capital K_{t+1} at the end of period t for production in period $t+1$, which they can sell on the market without adjustment cost at the end of the subsequent period. The capital choice problem of the intermediate goods firms is therefore static.To fund capital, the firm issues equity S_t at a price Q_t such that $Q_t K_{t+1} = Q_t S_t$.[10] Capital is mainly funded by banks who purchase capital securities S_t^B but the central bank also acquires capital securities S_t^{CB} to offset CBDC issuance on the asset side of its balance sheet, such that in total $S_t = S_t^B + S_t^{CB}$. Production in the intermediate goods sector is given by

$$Y_t^M = A_t \left(U_t \xi_t K_t \right)^{\alpha} L_t^{1-\alpha} \tag{10}$$

with A_t denoting total factor productivity and α the output elasticity of capital.

$$R_{t+1}^S = \left(\alpha \frac{P_{t+1}^M Y_{t+1}^M}{\xi_{t+1} K_{t+1}} + Q_{t+1} - \delta(U_{t+1}) \right) \frac{\xi_{t+1}}{Q_t} \tag{11}$$

where P_t^M denotes the price of the intermediate good.

The retail goods firm operates in a monopolistic competition setting, which consists of a continuum of mass unity of differentiated retail firms, indexed by R, using intermediate output as the sole input with a 1:1 input output ratio i.e. $Y_t^M = Y_t^R$. The final aggregated output on the CM, Y_t^{CM}, is a constant elasticity of substitution (CES) aggregate of the output produced by retailer R, Y_t^R, and is given by

$$Y_t^{CM} = \left[\int_0^1 (Y_t^R)^{\frac{\epsilon-1}{\epsilon}} dR \right]^{\frac{\epsilon}{\epsilon-1}} \tag{12}$$

[9]The functional form of the depreciation rate of capital is set to $\delta(U_t) = \delta_c + \frac{b}{1+\zeta} U_t^{1+\psi}$, with $b = \alpha P^{M,SS}/K^{SS}$ and $\delta_c = \delta_{SS} - b/(1+\zeta)$ and the subscript "SS" indicated the respective steady state values.

[10]There are no credit frictions in the funding of the intermediate goods firm. The credit friction in the model relates to the funding the bank obtains from the household.

where ϵ denotes the elasticity of substitution between goods. As retailers can only reset their price with probability $1-\gamma$ each period, the aggregate price level on the CM is sticky and evolves according to

$$P_t^{CM} = \left\{(1-\gamma)(P_t^*)^{1-\epsilon} + \gamma\left[(1+\pi_{t-1}^{CM})^{\gamma_P} P_{t-1}^{CM}\right]^{1-\epsilon}\right\}^{\frac{1}{1-\epsilon}} \tag{13}$$

Before discussing policy, we first turn to describing the DM and its interlinkages with the CM.

2.2 The decentralised market

At the beginning of the DM, a fraction σ of households receive a business opportunity and become shop owners (sellers), whereas a fraction σ become buyers and a fraction $(1-2\sigma)$ do not participate in the DM[11]. The value function in the DM before the realisation of the taste shock is therefore a weighted average of the different roles:

$$E_t(V_t^{DM}) = \sigma V_t^{DM,B} + \sigma V_t^{DM,S} + (1-2\sigma)\beta E_t(V_{t+1}^{CM}) \tag{14}$$

where $V_t^{DM,B}$ and $V_t^{DM,S}$ denote the value of being a buyer or a seller in the DM and V_{t+1}^{CM} reflects the value of participating in the next CM, adjusted with the households' discount factor between periods β. In order to complete the household problem and the optimality conditions given in equation 3, we need to determine the marginal value of assets for buyers and sellers in the DM.

$$E_t(V_{t,a_t}^{DM}) = \sigma V_{t,a_t}^{DM,B} + \sigma V_{t,a_t}^{DM,S} + (1-2\sigma)\beta E_t(V_{t,a_t}^{CM}) \quad \text{for} \quad a = \{M_t, D_t, B_t\} \tag{15}$$

Buyers and sellers are matched one-to-one, i.e. we assume that an efficient matching technology exists in the DM that matches each buyer with a corresponding seller. With respect to the pricing mechanism, we assume price taking as introduced into monetary search models by Rocheteau and Wright (2005). This mechanism is more tractable than alternative pricing mechanisms such as bargaining or price posting with directional search (see Gu and Wright (2016) for additional pricing mechanisms). It keeps the double coincidence and the verifiability assumptions that make money essential in trading on the DM, although the price is set at the market-clearing level by a Walrasian auctioneer. We believe that this is a more realistic description of a retail market than bargaining that is used in many search-based monetary models[12]. A buyer derives concave utility $U(C_t^{DM})$ from consuming the DM good C_t^{DM}. As money is essential in the DM, a preference shock to DM consumption translates into a shock to the demand for means of payment. The

[11] For a detailed derivation of the household problem see the appendix A.1.

[12] A different pricing mechanism would impact DM allocations. However, in Aruoba and Schorfheide (2010) using price taking or bargaining to determine the terms of trade does not significantly change the model implications and empirical performance.

buyers face the following optimisation problem:

$$V_t^{DM,B} = \max_{C_t^{DM}, m_t, d_t} \left\{ U(C_t^{DM}) + \beta E_t V_{t+1}^{CM}(M_t - m_t, D_t - d_t, B_t) \right\} \tag{16}$$

with $U(C_t^{DM}) = \Psi ln(C_t^{DM})$ s.t. to the constraints

$$\begin{aligned} & P_t^{DM} C_t^{DM} = m_t + d_t, \\ & 0 \leq m_t \leq \eta_M M_t, \\ & 0 \leq d_t \leq \eta_D D_t \end{aligned} \tag{17}$$

Buyers aim to maximise utility from DM consumption which we assume to be of the same functional form as CM consumption, where Ψ_t captures the relative weight of DM consumption compared to CM consumption. Ψ_t can also be subject to an aggregate preference shock. Choosing DM consumption, households take into account the resulting lower money balances in the subsequent DM. Consumption is paid with CBDC and deposit transfers m_t and d_t. Further, transactions are constrained by the liquid money balances ($\eta_M M_t$ and $\eta_D D_t$) that were brought into the DM and no negative money holdings are possible. Only a fraction $0 \leq \eta_M \leq 1$ and $0 \leq \eta_D \leq 1$ of CBDC and deposits can be spend in the DM. The limit on liquid assets that can be spent on DM consumption is used to account for relatively inelastic fixed spending such as rent, insurance or utilities.[13]

We can derive the demand for the DM consumption good C_t^{DM} from the optimality conditions for the buyer, which implies that the demand for goods in the DM satisfies:

$$U_{C_t^{DM}} = P_t^{DM} \left[\beta \varrho_{t+1}^{CM} R_{t+1}^M + (\lambda_t^{m,h} - \lambda_t^{m,l}) \right] \tag{18}$$

where $\lambda_t^{m,l}$ and $\lambda_t^{m,h}$ are the Lagrange multipliers associated with the lower and upper constraints on CBDC holdings, i.e. on $0 \leq m_t \leq \eta_M M_t$. Buyers thus consume the DM good C_t^{DM} until their marginal utility of consumption equals the relative price of the DM good, P_t^{DM} multiplied by the marginal benefit of holding money that is described by the expression in brackets, namely by the value of money balances carried over into the next CM and the liquidity value of money in the DM, expressed by the Lagrange multipliers that we will discuss in more detail below. Further, the buyer's optimality conditions (equation 19) yields that the difference in the interest rate between CBDC and money reflects the difference in the shadow value of holding CBDC and deposits, where $\lambda_t^{d,l}$ and $\lambda_t^{d,h}$ are the Lagrange multiplier associated to deposit holdings:

$$(R_{t+1}^D - R_{t+1}^M) = \frac{(\lambda_t^{m,h} - \lambda_t^{m,l}) - (\lambda_t^{d,h} - \lambda_t^{d,l})}{\beta \varrho_{t+1}^{CM}}. \tag{19}$$

[13] Such modelling device also finds application in e.g. Rocheteau et al. (2018) or Aruoba and Schorfheide (2010).

The sellers' optimisation problem (equation 20) takes into account costs of producing the DM $\mathcal{C}_t^{DM}$ good as well as the benefits of having more money balances in the subsequent CM. For tractability, we model production of the DM output to be simply a concave function of effort exerted by the seller. Thus, in contrast to the CM, DM production does not require capital. This results in convex production costs for the DM good which we assume to take the form $\mathcal{C}(C_t^{DM}) = \nu(C_t^{DM})^{\frac{1}{\nu}}$.

$$V_t^{DM,S}(\cdot) = \max_{C_t^{DM}} \left\{ -\mathcal{C}(C_t^{DM}) + \beta E_t V_{t+1}^{CM}(M_t + m_t, D_t + d_t, B_t) \right\} \tag{20}$$

From the optimality conditions of the sellers, we can derive the supply function which states that the relative price of the DM good equals the marginal cost of production $\mathcal{C}_{C_t^{DM}}$, relative to the discounted marginal benefit of money in the CM.

$$P_t^{DM} = \frac{\mathcal{C}_{C_t^{DM}}}{\beta \varrho_{t+1}^{CM} R_{t+1}^M} \tag{21}$$

Combining the buyers' demand and the sellers' supply function this yields the DM equilibrium condition:

$$\frac{U_{C_t^{DM}}}{\mathcal{C}_{C_t^{DM}}} = 1 + \frac{\lambda_t^{m,h} - \lambda_t^{m,l}}{\beta \varrho_{t+1} R_{t+1}^M} \tag{22}$$

We now discuss how money holdings affect the outcome in the DM. Depending on the available money balances, two regimes can emerge in the DM: If there is enough liquidity, transaction constraints related to money holdings do not bind (i.e. $\lambda_t^{m,l} = \lambda_t^{m,h} = \lambda_t^{d,l} = \lambda_t^{d,h} = 0$) and the optimal output on the DM equates the marginal utility of consumption with the marginal costs of production of the DM good:

$$U_{C_t^{DM*}} = \mathcal{C}_{C_t^{DM*}} \tag{23}$$

In this case, DM output does not depend on CBDC and deposit balances. This implies that there is no transaction value of an additional unit of money in the DM and the interest rates on CBDC, deposits and bonds are equal with $R_{t+1}^M = R_{t+1}^D = R_{t+1}^B = \frac{\varrho_t^{CM}}{\beta \varrho_{t+1}^{CM}}$.

Money balances however influence DM market outcomes if liquidity is not abundant which is also reflected in the interest rates. In the constrained case, we can establish that, as long as CBDC and money balances are positive and DM consumption is valued by buyers, only the upper constraints on CBDC and deposits will bind. Further, if one upper constraint is binding, then both upper constraints of CBDC and deposits will be binding (see appendix A.1). In this way, the Lagrange multipliers $\lambda_t^{m,h}$ and $\lambda_t^{d,h}$ reflect the liquidity value of a marginal unit of money and is represented in the marginal value of CBDC and deposits:

$$
\begin{aligned}
V^{DM}_{t,M_t} &= \beta \varrho^{CM}_{t+1} R^M_{t+1} + \sigma \eta_M \lambda^{m,h}_t = \varrho^{CM}_t \\
V^{DM}_{t,D_t} &= \beta \varrho^{CM}_{t+1} R^D_{t+1} + \sigma \eta_D \lambda^{d,h}_t = \varrho^{CM}_t
\end{aligned}
\tag{24}
$$

The value of $\lambda^{m,h}_t$ is determined by the optimality condition of the buyer (equation 18) and the seller (equation 21)

$$
\lambda^{m,h}_t = \beta \varrho^{CM}_{t+1} R^M_{t+1} \left[\frac{U_{C^{DM}_t}}{\mathcal{C}_{C^{DM}_t}} - 1 \right], \tag{25}
$$

whereas the value of $\lambda^{d,h}_t$ determined by equation 19. The constrained demand of the buyer is then determined by it's liquid money balances (equation 16)

$$
P^{DM}_t C^{DM} = \eta_M M_t + \eta_D D_t. \tag{26}
$$

.

We can now complete the optimality conditions for money holdings represented by the euler equations for CBDC and deposits:

$$
1 = \frac{\beta \varrho^{CM}_{t+1}}{\varrho^{CM}_t} R^M_{t+1} \left[(1 - \sigma\eta_M) + \sigma\eta_M \frac{U_{C^{DM}_t}}{\mathcal{C}_{C^{DM}_t}} \right] \tag{27}
$$

$$
1 = \frac{\beta \varrho^{CM}_{t+1}}{\varrho^{CM}_t} \left[(1 - \sigma\eta_D) R^D_{t+1} + \sigma\eta_D R^M_{t+1} \frac{U_{C^{DM}_t}}{\mathcal{C}_{C^{DM}_t}} \right] \tag{28}
$$

The value of an additional unit of money, enabling higher DM consumption, is thus captured by its liquidity premium on the interest rate. This expression weights the relative marginal utility of being able to increase DM consumption $\frac{U_{C^{DM}_t}}{\mathcal{C}_{C^{DM}_t}}$ with the probability of becoming a buyer in the DM σ and the fraction of money that can be used for spending η. Thus, the closer households get to their optimal DM consumption, the smaller the liquidity premium becomes. In this way there is a smooth transition to the euler equation in the unconstrained region in which $U_{C^{DM}_t} = \mathcal{C}_{C^{DM}_t}$.

2.3 Government and aggregation of DM and CM

The government has a fixed amount of government spending G. It further issues government bonds B_t, takes on any profits and losses from the central bank (T^{cb}_t) and taxes or pays transfers T_t households. The government budget is accordingly

$$
G + R^B_t B_{t-1} = T_t + B_t + T^{CB}_t \tag{29}
$$

Having set out the CM and the DM, we turn to aggregate real output and inflation resulting from the two sectors. First, note that the household optimisation problem is defined in real terms, with C_t^{CM} as numeraire, and M_t and D_t denote real CBDC and deposit holdings. Consequently, all gross rates of return (i.e. R_{t+1}^M, R_{t+1}^D and R_{t+1}^B) are also defined in real terms.

The combined real output of both markets is

$$\begin{aligned} Y_t &= Y_t^{CM} + Y_t^{DM} \\ &= C_t^{CM} + I_t + G_t + f(\cdot)(I_t^N + I_t^{SS}) + \sigma P_t^{DM} C_t^{DM} \end{aligned} \tag{30}$$

Output on the DM is adjusted with the relative price between the CM and the DM good, P_t^{DM}, and the probability to consume in the DM, σ. We follow Aruoba and Schorfheide (2011) in defining economy-wide inflation by a Fisher price index, weighting the size of each market with its steady state share:

$$\ln \pi_t = \ln \frac{P_t}{P_{t-1}} = (1 - s^{SS}) \ln \pi_t^{CM} + s^{SS} \ln \pi_t^{DM} \tag{31}$$

where s^{SS} is the steady state share of the DM market and $\pi_t^{DM} = P_t^{DM}/P_{t-1}^{DM}$. The aggregate DM and CM price level would therefore evolve according to

$$P_t = P_0 \Pi_{\tau=1}^t \left(\pi_\tau^{CM}\right)^{1-s^{SS}} \left(\pi_\tau^{DM}\right)^{s^{SS}}$$

2.4 Central Bank

The central bank has two policy instruments. It sets the interest rate on government bonds and issues a CBDC. In this way, it can use the two instruments to target the different functions of money. It affects the store of value function of assets by setting the rate on government bonds, whereas it steers the medium of exchange function by setting the interest rate on CBDC.

The central bank sets the nominal interest rates on public debt/government bonds via a standard Taylor rule in which it takes into account output and inflation dynamics in both sectors.

$$i_{t+1}^B = i^{B*} + \kappa_\pi \pi_t + \kappa_y (\log Y_t - \log Y_t^*) + \varepsilon_t^i \tag{32}$$

Real and nominal interest rates are connected by the Fisher equation $(1 + i_{t+1}^a) = R_{t+1}^a (1 + E_t \pi_{t+1})$ for $a = \{M, D, B\}$. CBDC is issued following the interest rate rule of the form

$$i_{t+1}^M = i_{t+1}^B - \kappa_m \left(M_t - \overline{m}\right) \tag{33}$$

With this rule, the central bank targets the liquidity premium, i.e. the difference between the bond and CBDC interest rate $i^b_{t+1} - i^M_{t+1}$, by its issuance of CBDC. If there is abundant liquidity in the market, the central bank just issues a (small) base amount of CBDC $M^S_t = \overline{m}$. In this case, there is no liquidity premium and $i^M_t = i^B_t$. However, if liquidity is constrained and demand for CBDC M_t rises, the central bank satisfies demand, however at an increasingly unattractive interest rate. Thus, the more CBDC demand surpasses the base issuance $(M_t - \overline{m})$, the lower i^M_{t+1} becomes. This has two implications. Firstly, it automatically stabilises the demand for CBDC by making CBDC less interesting, the higher CBDC demand is. It thereby reflects the proposals of a tiered remuneration approach (e.g. see Bindseil, 2020) in a continuous way. Simultaneously, it allows a certain liquidity premium in times where deposits are low (indicating stressed banking sector), and thereby stabilises the situation of banks by easing of funding conditions.

To offset CBDC issuance on the asset side of its balance sheet, the central bank purchases capital securities S^{CB}_t in the amount of CBDC in circulation.[14]

$$M_t = Q_t S^{CB}_t \tag{34}$$

All profits/ losses of the central bank are distributed via lump-sum transfers T^{CB}_t to the government. The budget balance of the central bank is accordingly:

$$R^M_t M_{t-1} + Q_t S^{CB}_t + T^{CB}_t = M_t + R^S_t Q_t S^{CB}_{t-1} \tag{35}$$

3 Simulations

In this section, we analyse different types of shocks that can be classified as business cycle shocks, financial shocks and shocks to the means-of-exchange function of CBDC. For the business cycle and financial shocks, we first compare the dynamics of our New Monetarist/New Keynesian model (henceforth NMNK) with those arising in Gertler and Karadi (2011) (henceforth GK11). For that exercise, we first assume that CBDC does not exist and bank deposits are the only means of exchange on the DM, which allows us to assess the difference in the dynamics that results from including a DM in the model. We find qualitatively similar impulse responses, although the amplitude of the response of output and deposits to the business cycle shocks and the capital quality shock tends to be somewhat reduced in our model compared to GK11. Responses to a shock to bank net worth are very close in both models, whereas a monetary policy shock has a larger effect on output in the NKMK model compared to GK11. In general, the size of the DM determines the extent to which impulse responses differ in the NMNK model compared

[14] Alternatively, the central bank could offset CBDC issuance by granting credit to banks as e.g. in Brunnermeier and Niepelt (2019) or Gross and Schiller (2021) or by purchasing government securities as in Barrdear and Kumhof (2021) or Kim and Kwon (2019).

to GK11 (see also the discussion of the parameter calibration in Section 3.1.

After having established that the addition of a DM to the GK11 model does not fundamentally change the model dynamics, we analyse in a second step how the existence of CBDC changes the dynamic responses to these shocks. We find that, in general, the existence of CBDC reduces the persistence and amplitude of the responses of output and inflation to business cycle and financial shocks. The reason is that the central bank through its provision of CBDC now can influence the liquidity premium, i.e. the difference between the yields on bonds and deposits, which reflects the value of deposits as a means of exchange on the DM. The liquidity premium on the one hand affects consumption on the DM; on the other hand, it determines bank funding conditions and has an effect on investment and output through this channel. We will discuss transmission through these different channels in more detail in Sections 3.2 and 3.3.

Shocks to the medium of exchange function do not exist in GK11 and therefore can only be analysed in the NMNK model. We consider a shock to the supply of CBDC, a shock to the preference for consumption on the DM, which can be interpreted as a shock to money demand, as well as a shock to the usefulness of bank deposits as a means of payment. By comparing reactions in macroeconomic outcomes to these shocks with and without the existence of CBDC, we get further insights into the transmission channels and the role of CBDC as an additional monetary policy instrument in the NMMK model. Like in George et al. (2020), we find that the existence of CBDC allows the central bank to target fluctuations in the liquidity premium and thereby opens up a second channel through which monetary policy can influence output and inflation.

3.1 Calibration and shocks

Table 1 documents the parameter values for the numerical simulation. We calibrate the model such that one period equals a quarter. With a few exceptions we adopt most parameter values from Gertler and Karadi (2011) for the CM. Tractability of the NMNK model with a DM and a CM requires a unitary labour elasticity. Although this affects the results quantitatively, we checked that it does not qualitatively change the pattern of the impulse responses. As this rules out an increasing marginal cost of labour, we follow Aruoba and Schorfheide (2011), who feature the same set-up with a unitary labour elasticity, and align the utility weight of labour with their estimated value of 24.3 (compared to a value of 3.409 in GK11). In addition, we calibrate the share of government spending G and investment adjustment costs ι to match euro area data by setting government spending over steady state output to 48%, which is the average share of total government expenditure to GDP for euro area countries between 2004-2021. To better fit the investment adjustment costs to euro area values, we take the value of Smets and Wouters (2003) of $\iota = 5.991$. We further set the probability of keeping prices fixed to 0.82, aligning it with the value in the ECB's New Area Wide Model, see Coenen et al.

(2018).

The calibration of the DM is informed by euro area payment data. We set the share of households engaging in the DM to $2\sigma = 0.4$ which results in a share of DM consumption to total consumption of 51%; roughly equal to the 55% share of transactions at the point of sale and from peer to peer in the euro area (European Central Bank, 2020). The relative utility weight of DM consumption is set to $\Psi = 5.5$ and the payment fraction of both deposits and CBDC to $\eta_D = \eta_M = 1/3$ to target a liquidity premium of 1.4% p.a. in the model version without CBDC. This is in line with the average difference between the ECB's interest rate on the main refinancing operations (MRO) and the rate on overnight deposits in the euro area between 2000-2016,[15] which we use as a proxy for the liquidity premium. We target a steady state share of CBDC to total money (CBDC and deposits) of 15.8% which is the 2000-2016 average euro area share of currency in circulation to M1. To achieve this, we calibrate the base issuance of CBDC in the absence of a liquidity premium (i.e. for the unconstrained DM) to be very small with $\bar{m} = 0.001$ and the CBDC interest rate reaction to the liquidity premium to $\kappa_m = 0.018$. These parameter values result in a steady state share of CBDC to total money of 15.7%.

We analyse eight different types of shocks that we group into three categories: business cycle shocks, financial shocks and shocks to the means-of-exchange function of money. To investigate business cycle dynamics, we study a supply and a demand shock, modelled as a 1% shock to total factor productivity (TFP), A, as in GK11 and as a shock to the discount factor β of 0.5%. For the financial shocks, which are in line to the analysis in GK11, we examine a deterioration in the quality of capital ξ of 5%, a 1% shock to banks' net worth N_t^e and a conventional monetary policy shock of 1% to the bond interest rate rule i_t^B. Finally, we look at shocks affecting money as a means of exchange by analysing an increase to the preference for DM consumption Ψ of 5%, a disturbance to the usefulness of deposits for payments (i.e. a payment technology shock to η_D) of 5% and a shock to the CBDC interest rate rule i_t^M of 0.5%. All shocks, except the bank net worth shock, follow an AR(1) process in the form of $e_{i,t} = \rho_i e_{i,t-1} - \epsilon_{i,t}$ for shock type i, $i = \{A, \beta, \xi, i^B, \Psi, i^M\}$.[16]

Parameter	**Value**	**Description**
Households		
β	0.990	Discount factor
χ	24.3	Relative utility weight of labour
Ψ	5.5	Relative utility weight of DM consumption
σ	0.2	Probability of being a buyer/seller in the DM

Continued on next page

[15] We discard data from March 2016 on as the MRO rate equalled zero from then on.

[16] As the Taylor rule in our model is without interest rate smoothing for tractability reasons, we account for persistence in the monetary policy shock process.

ν	0.68	Effort elasticity of DM good production
η_D	0.3323	Share of deposits available for DM transactions
η_M	0.3323	Share of CBDC available for DM transactions
Banks		
θ	0.972	Survival probability of bankers
λ	0.381	Fraction of capital that can be diverted
ω	0.002	New banks' endowment fraction
Firms		
α	0.33	Elasticity of capital demand
ι	5.991	Scaling parameter investment adjustment costs
ζ	7.2	Elasticity of marginal depreciation w.r.t. utilisation rate
ϵ	4.167	Elasticity of substitution between goods
γ	0.82	Probability of keeping prices fixed
γ_p	0.241	Price indexation of inflation
Policy parameters		
κ_π	1.5	Taylor rule inflation coefficient
κ_y	0.5/4	Taylor rule output coefficient
κ_M	0.018	CBCB rule coefficient
$\overline{m}$	0.001	CBDC base issuance
Shock parameters		
ρ_a	0.95	Persistence of TFP shock
ρ_β	0.9	Persistence of discount factor shock
ρ_ξ	0.66	Persistence of capital quality shock
ρ_{N^e}	0	Persistence of net worth shock
ρ_{i^B}	0.9	Persistence of monetary policy shock
ρ_Ψ	0.96	Persistence of DM preference shock ((Aruoba et al., 2011)
ρ_{i^M}	0.9	Persistence of CBDC rule shock

Table 1: Parameter values

3.2 Business cycle shocks

We first document that our NMNK model produces qualitatively similar results to the GK11 setup when we do not allow for CBDC. While the CM is basically identical to GK11, the existence of a DM assigns an explicit role to the means-of-exchange function of deposits and therefore affects the way how business cycle shocks are transmitted. Figure 2 compares the responses to a negative 1% TFP shock from the NMNK model without CBDC to GK11. A decline in TFP reduces capital, output and consumption on the CM. Although there is no genuine production on the DM, the productivity shock spills over to the DM as a part of the output is traded decentrally, leading also to a decline in DM consumption. At the same time, DM consumption becomes more costly, leading to a lower demand for deposits and thereby reducing the liquidity premium. The decrease in

the liquidity premium reduces the demand for deposits and thereby affects consumption demand on the DM. Overall, this reduces the relative price of DM consumption and shifts some CM consumption to the DM, thereby mitigating the decline in consumption on the DM but amplifying it on the CM. While the consumption response is initially somewhat larger than in GK11, the subsequent dynamics are similar. As deposits are needed for transactions on the DM, they decline less than in GK11 despite the decline in the liquidity premium, which means rising funding costs for banks. The capital premium and gross investment react less than in GK11 on impact, leading to a less pronounced output response. Inflation, however, is dominated by the strong price increase on the CM and shows a higher and more persistent reaction to the TFP shock than in GK11.

Figure 3 shows the effects of a shock to the discount factor β with a persistence of 0.9, which we interpret as a demand shock. The liquidity premium rises, reflecting the increased preference for consumption that affects both the CM and the DM. Relative DM prices rise through increasing marginal costs of DM production, the higher discounting of the future and lower deposit interest rates with a higher liquidity premium. As deposits serve as a medium of exchange and are essential for consumption on the DM, they decline much less in the NMNK model than in GK11. The high demand for deposits preserves bank funding and financial intermediation, which cushions the decline in investment. While in GK11 the fall in investment exceeds the increase in consumption and leads to an overall decline in output, in the NMNK model the additional role of deposits as a means of exchange results in a moderately positive output response with a considerably higher inflation path.

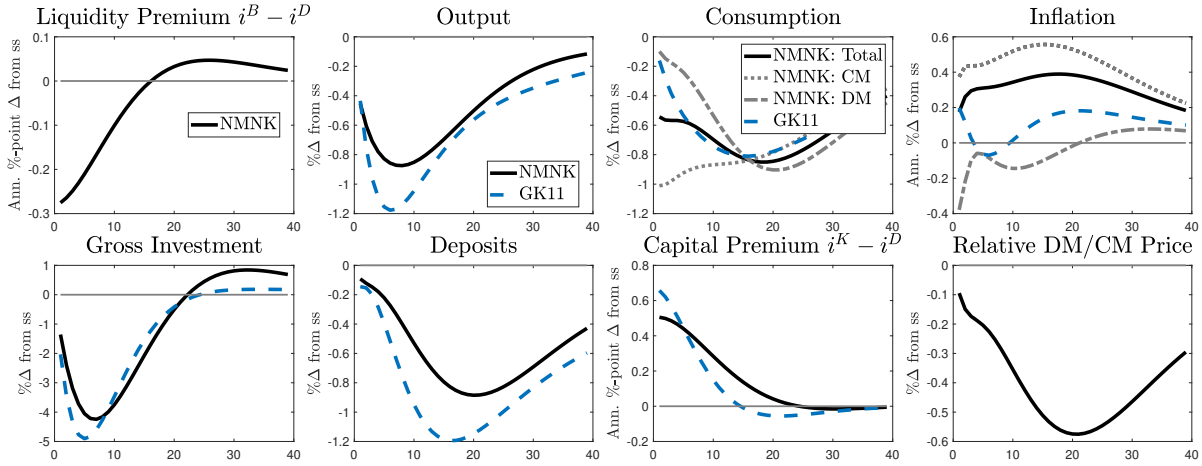

Figure 2: Responses to a 1% TFP shock in NMNK without CBDC and GK11 models

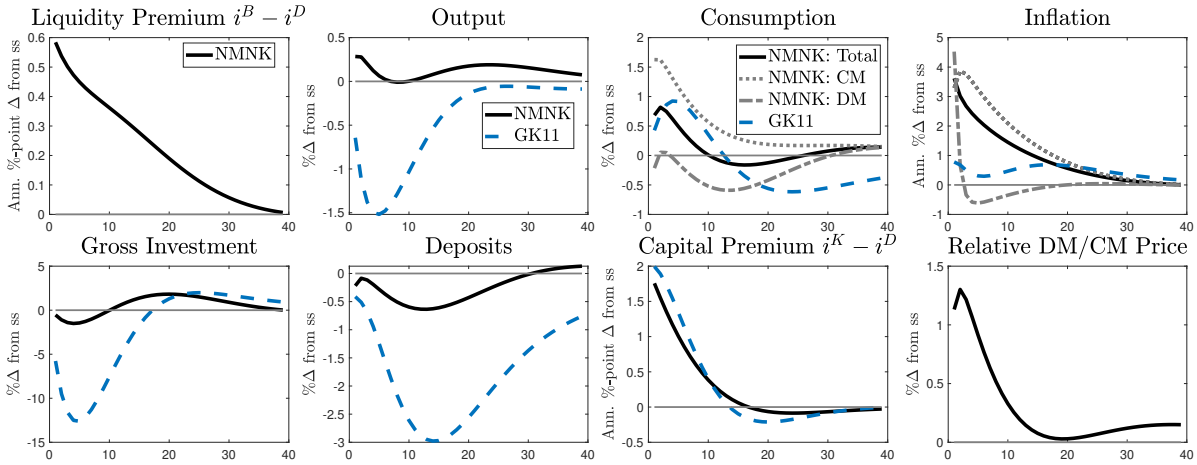

Figure 3: Responses to a 0.5% discount factor shock in NMNK without CBDC and GK11 models

Summing up, we find that the NMNK model broadly displays similar dynamics to business cycle shocks as the GK11 model. Through the addition of a DM, the liquidity premium obtains a central role in trading off the means-of-exchange function of deposits versus their role in bank funding and intermediation, which in turn affects investment. Compared to GK11, we obtain a somewhat muted output response as investment tends to react less strongly to supply and demand shocks, whereas inflation displays more elevated and persistent dynamics. We now move on to study how the existence of CBDC as an additional means of exchange modifies the responses to supply and demand shocks in the NMNK model.

Figure 4 shows the response to the TFP shock for the NMNK model with and without CBDC.[17] The CBDC rule implies that the central bank stabilises fluctuations in the liquidity premium in reaction to a change in payment conditions resulting from the shock. While in the model without CBDC the endogenous reaction of the liquidity premium shifts some consumption from the CM to the DM, the stabilisation of the liquidity premium limits this shift. On the one hand, the drop in DM consumption with CBDC is cushioned less, with the decline in consumption on impact being larger but recovering faster. On the other hand, a less pronounced drop in the liquidity premium leads to an increase in deposits, implying better and more stable funding conditions for banks, which contributes to a smaller and less persistent decline of investment. On balance, the decline in output is similar with and without CBDC on impact but recovers more quickly with a CBDC. Inflation with a CBDC is more elevated in the first few quarters but is less persistent and lower thereafter.

Figure 5 depicts the response of the NMNK model with and without CBDC to

[17]Note that the dashed lines in Figure 4 are identical with the solid lines in figure 2.

the discount factor shock.[18] The higher discounting of the future increases consumption demand. Deposits, limited by the leverage constraint, cannot satisfy this demand leading to a rise in the liquidity premium. A CBDC satisfies the liquidity demand, but to less attractive conditions, and thereby enables higher DM consumption and stabilises the liquidity premium. But this also limits the reduction in funding costs for banks, leading to a decline in deposits and a longer contraction in investment. The countervailing effects of consumption and investment lead to an initially higher increase in output but after an undershooting to a somewhat slower recovery driven by the investment response. Inflation is slightly lower and less persistent.

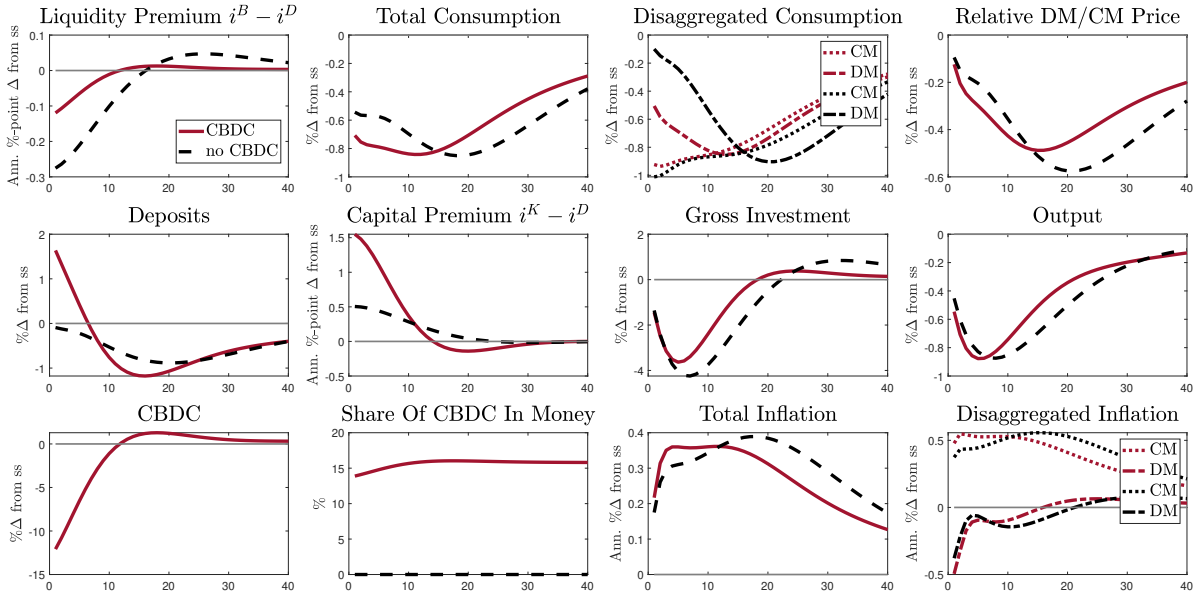

Figure 4: Responses to a 1% TFP shock in the NMNK model with and without CBDC

[18]The dashed lines in Figure 5 are identical with the solid lines in figure 3.

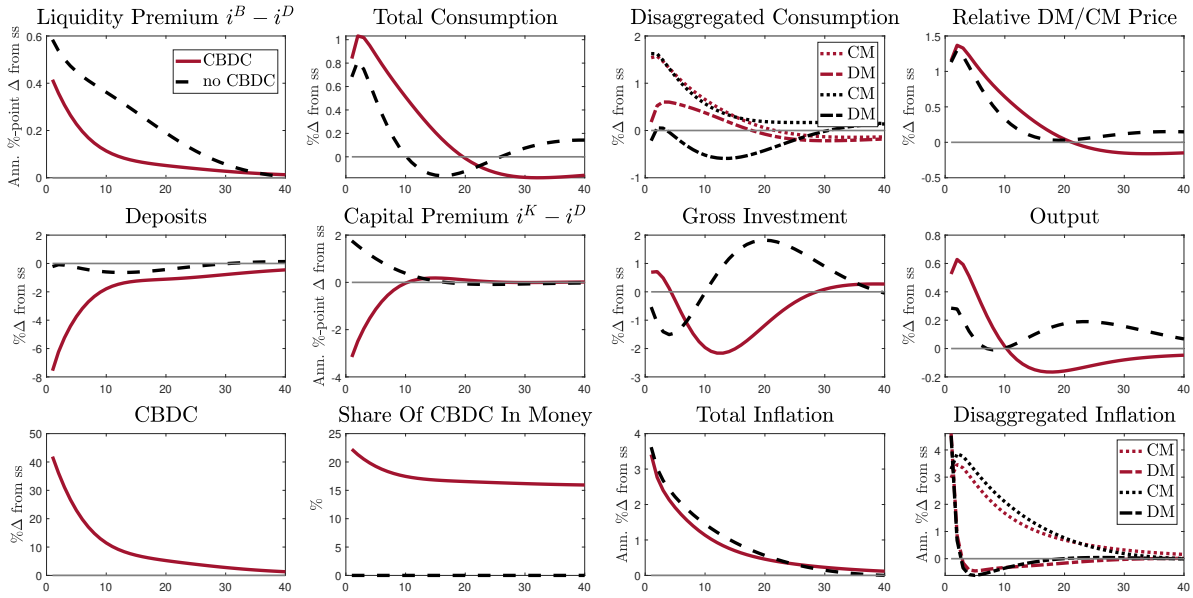

Figure 5: Responses to a 0.5% discount factor shock in NMNK model with and without CBDC

3.3 Financial shocks

This section analyses financial shocks, namely a capital quality shock, a shock to banks' net worth and a monetary policy shock, by first comparing the dynamics of the NMNK model to those of GK11 and then assessing how the existence of a CBDC affects the transmission in the NMNK model. Overall, we find that with a CBDC the response of output and inflation to financial shocks tends to be dampened when CBDC is available as an additional means of payment. Figure 6 depicts the response to a 5% drop in capital quality that, like in GK11, demonstrates the amplification of business cycle fluctuations through financial frictions. The shock lowers the price of capital securities and affects the asset side of banks' balance sheet. Due to the decreased effective capital stock, output, investment and consumption fall on impact. The shock tightens banks' leverage constraint, thereby forcing them to reduce deposits to deleverage. As in the NMNK model deposits are needed as a means of exchange in the DM, their volume falls less than in GK11, which reduces the decline in investment and output compared to GK11. The lower consumption on the DM nonetheless reduces the transaction demand for deposits even more than the fall in deposits induced by banks' deleveraging. Therefore, the liquidity premium declines initially but overshoots later, together with the overshooting of investment. Since output falls less than in GK11, inflation increases and returns more gradually to its steady-state.

Figure 7 displays the responses to a 1% shock to banks' net worth, which is redistributed to households like in GK11. This experiment demonstrates the effects of the

financial friction in the CM as it affects the liabilities side of banks' balance sheet without reducing overall wealth in the economy. In a frictionless model, this redistribution should have no effects but in GK11 and the NMNK model this shock leads to contraction in investment and output. The liquidity premium declines but – as the shock has affected the liabilities side of banks' balance sheet and not the asset side like the capital quality shock – deposits increase driven by a substitution effect. While the consumption response on the CM is quantitatively similar to GK11, the higher stock of deposits raises consumption on the DM, resulting in an overall increase in consumption. As investment starts picking up over time, deposits fall, the liquidity premium recovers and consumption is reduced. Inflation falls less than in GK11 on impact, driven by the positive consumption response and then like in GK11 overshoots before converging back to its steady state value.

The third financial shock we investigate is a standard monetary policy shock to the central bank's interest rate (Taylor) rule. Responses to this shock, which in our set-up affects the bond interest rate, are shown in figure 8. The increase in the bond rate decreases the relative price of DM consumption, leading to a lower liquidity premium that, in addition to the higher bond rate, strains financial intermediaries through higher funding costs. Compared to GK11, the decline in investment, consumption and output as well as the initial impact on inflation are amplified. This exercise thus shows that traditional monetary policy in the NMNK model remains effective, with the existence of a DM without CBDC even strengthening the impact of the bond rate on the economy.

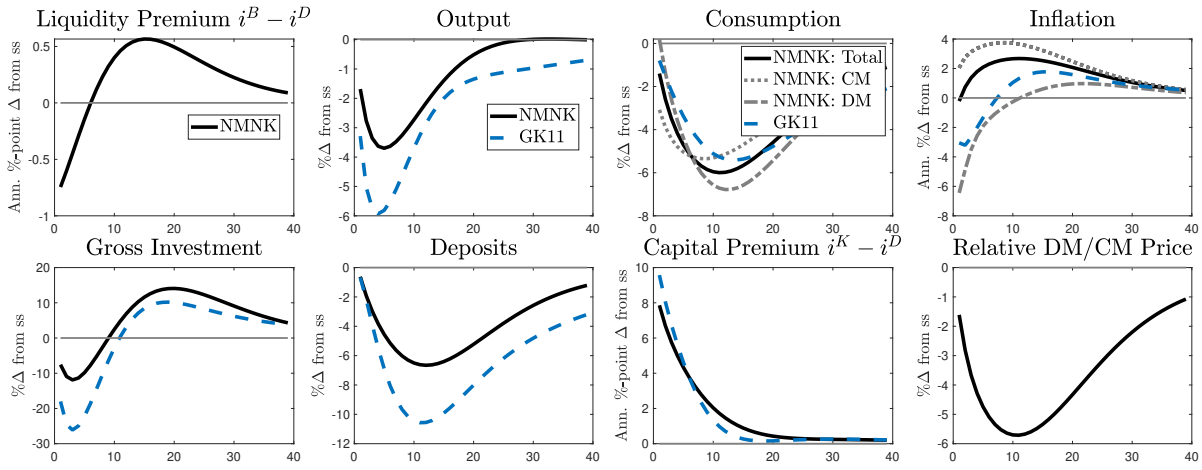

Figure 6: Responses to a negative 5% capital quality shock in NMNK without CBDC and GK11 models

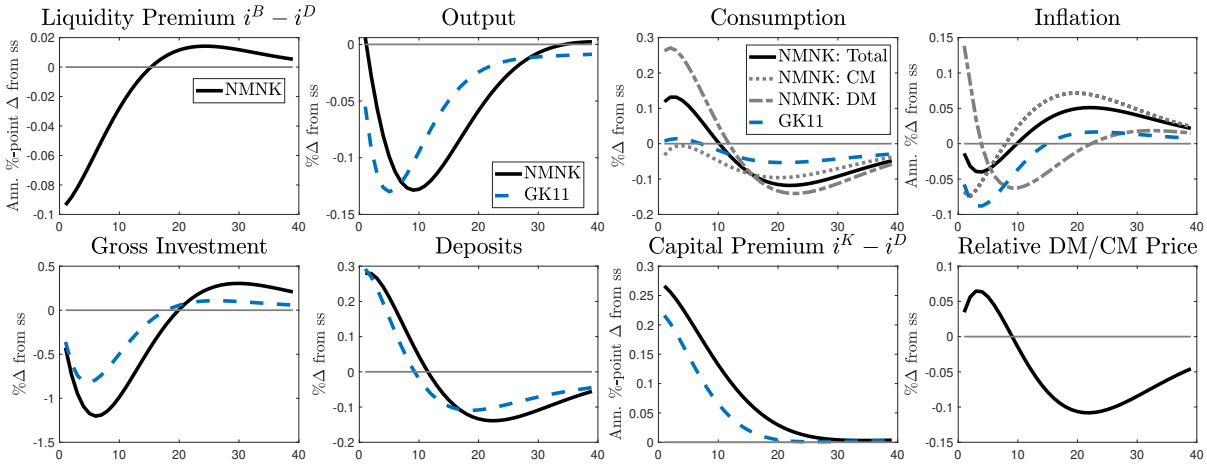

Figure 7: Responses to a negative 1% bank net worth shock in NMNK without CBDC and GK11 models

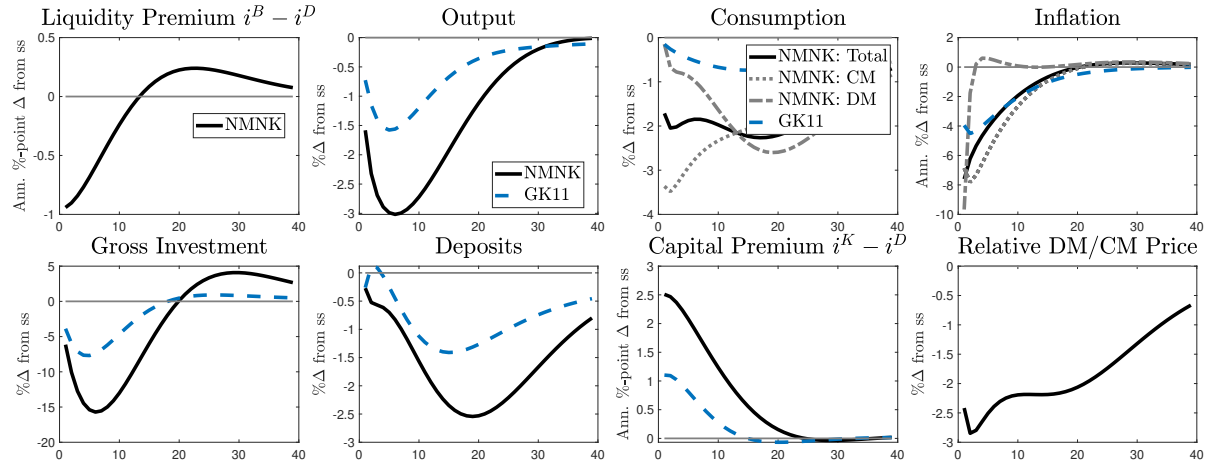

Figure 8: Responses to a contractionray 1% monetary policy shock in NMNK without CBDC and GK11 models

In sum, Figures 6 to 8 document that responses to financial shocks are similar in the NMNK and the GK11 models, although the existence of deposits as a means of payment introduces additional transmission channels that work through banks' balance sheets and the means-of-payment function of deposits. In particular, the strength of the response in investment and consumption depends on the type of shock and can either dampen or amplify the reaction of output and inflation compared to the original GK11 setup. Having established the overall similarity of the NMNK and the GK11 models, we investigate next how transmission in the NMNK model is affected by the existence of a CBDC.

The presence of CBDC with a remuneration policy that actively reacts to changes in the liquidity premium reduces output and inflation volatility in response to financial

shocks. Figure 9 shows the response to a decline in capital quality with and without CBDC. By stabilising the liquidity premium the central bank induces a larger drop in deposits than without CBDC as households substitute deposits for CBDC. At the same time, the capital premium increases in response to the capital quality shock. Taken together, these countervailing effects lead to slightly less volatility in the investment response that translates into a less pronounced albeit more persistent drop in output. With a CBDC, the inflation response to a capital quality shock is less volatile and less persistent.

Also for the bank net worth shock, the existence of a CBDC helps to contain fluctuations in output and inflation, as shown in Figure 10. The smaller drop in the liquidity premium supports a larger increase in deposits in the model version with CBDC. This eases bank funding, although a stronger increase in the capital premium counteracts this effect. As the increase in deposits results in an almost equivalent reduction in CBDC, total consumption increases less than in the case without CBDC, which contributes to stabilising the response of output. At the same time inflation dynamics are less pronounced, both on the DM as well as on the CM.

Figure 11 shows that the responses of output and inflation to a monetary policy shock with and without CBDC are broadly similar, which indicates that standard monetary policy would be equally effective with the existence of a CBDC. Yet, the responses imply that the strength of transmission through the various channels differs. Following a rise in the bond interest rate induced by the monetary policy shock, the reaction through the CBDC rule mitigates the drop in the liquidity premium. The existence of a CBDC also leads to a substitution of CBDC for deposits, shielding bank funding and increasing the capital premium, which stabilises the decline in investment. Consumption in the DM is affected by the endogenous decline in CBDC supply, which on balance results in an almost identical inflation response and a broadly similar response in output to a moneary policy shock regardless of the existence of CBDC. This analysis, however, assumes that the central bank does not employ a shock to the CBDC rule at the same time to better achieve its objectives.[19]

[19]We analyse a shock to the CBDC rule in Section 3.4

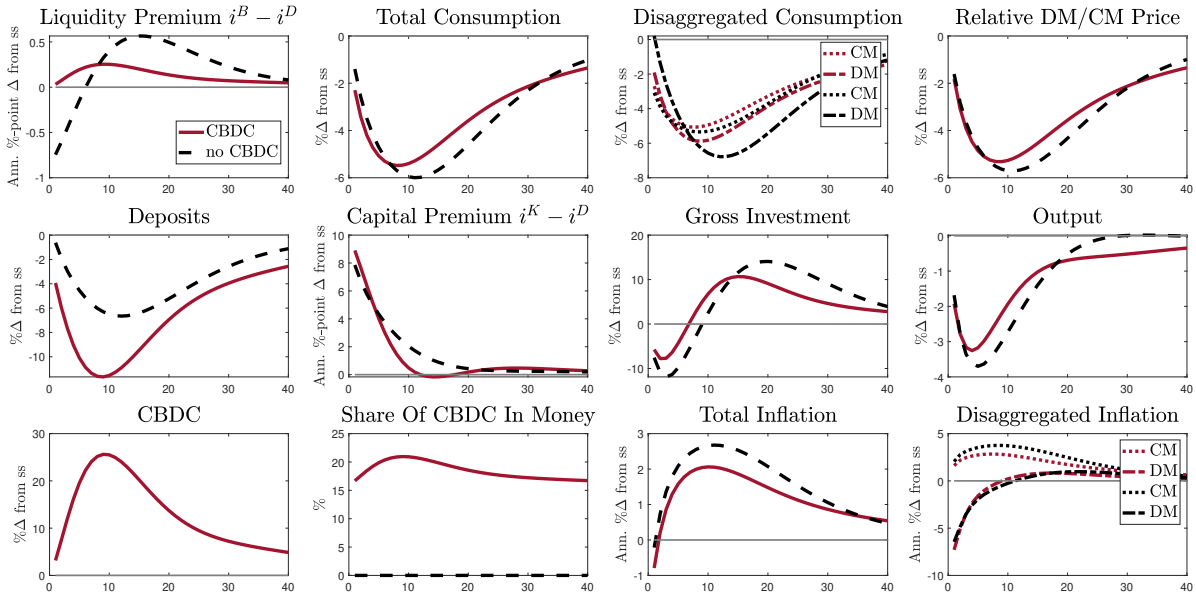

Figure 9: Responses to a negative 5% capital quality shock with and without CBDC

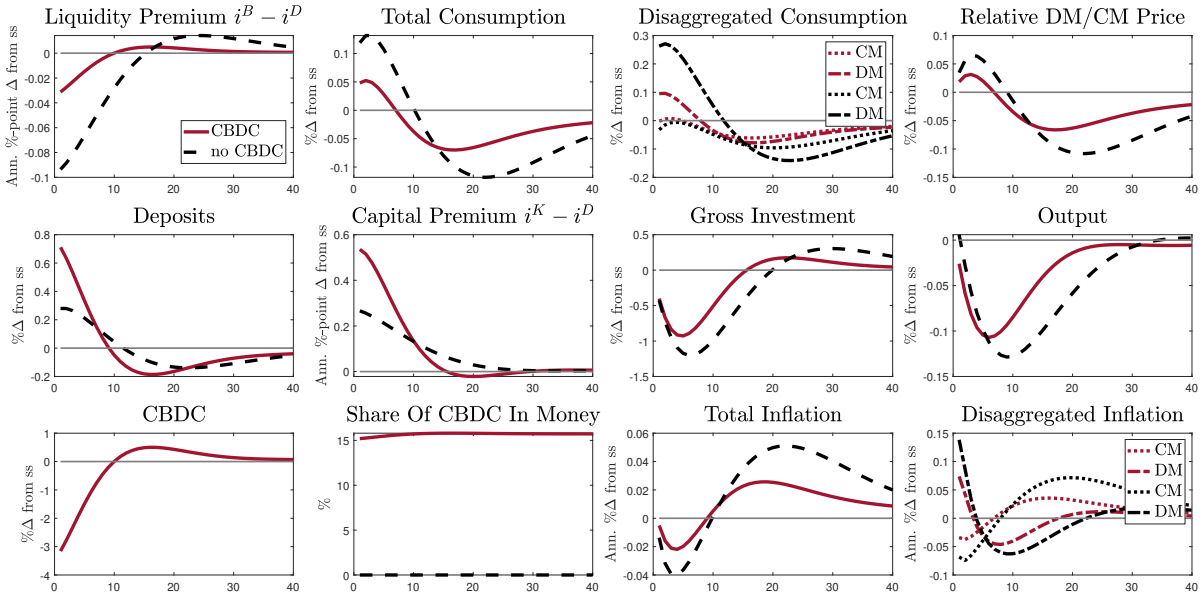

Figure 10: Responses to a negative 1% bank net worth shock with and without CBDC

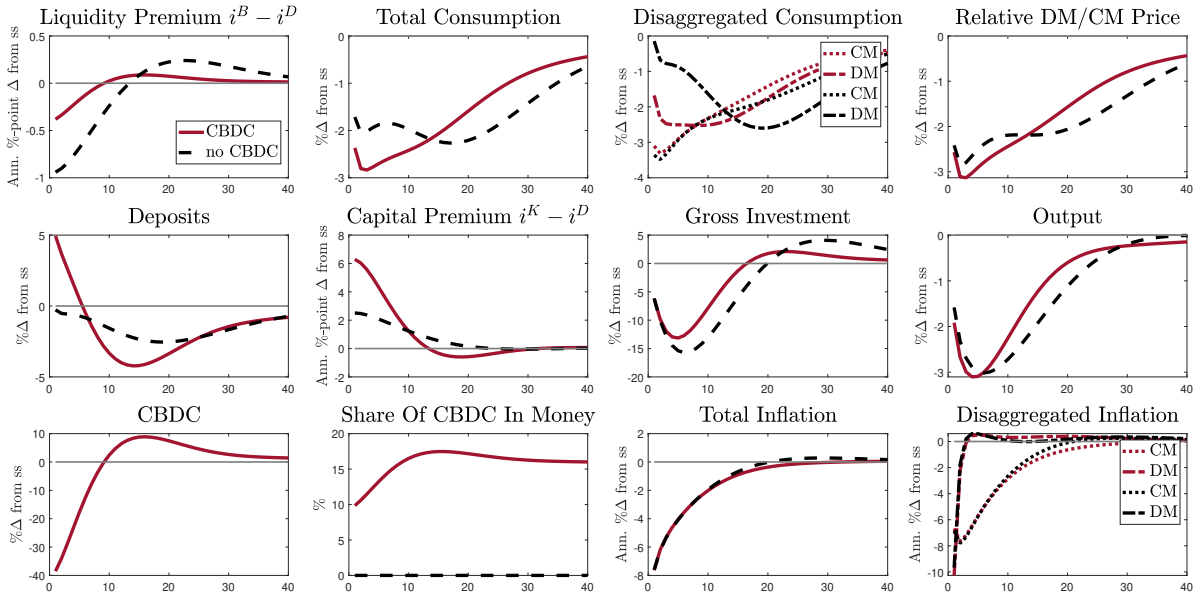

Figure 11: Responses to a contractionary 1% monetary policy shock with and without CBDC

3.4 Shocks to the medium-of-exchange function of money

Finally, we investigate shocks that are specific to the explicit modelling of the means-of-exchange function of money; a feature that is not present in the GK11 model. We therefore only compare the NMNK model responses without and with CBDC. Figure 12 shows the responses to a 5% increase in the preference of consuming the DM good. Aruoba et al. (2011) interpret this shock as a money demand shock, which increases the transaction demand for money. As banks' deposits issuance is limited by the leverage constraint, the liquidity premium increases, with the central bank satisfying a part of the increased demand for money in the model version with CBDC. Deposits fall by more in the version with CBDC as households substitute bank deposits for CBDC to some degree. Consumption on the DM increases with a higher supply of money, leading to an overall more pronounced increase in consumption with CBDC. As the central bank reduces the liquidity premium by supplying CBDC compared to the case without CBDC, bank funding conditions improve less in response to the money demand shock. The response of investment in the first few quarters after the shock is fairly similar as with a CBDC the negative effect of a smaller increase in the liquidity premium and more favourable funding conditions through a larger decline in the capital premium almost balance each other. Overall, with a CBDC output increases more but less persistently whereas inflation is less elevated and normalises faster than without a CBDC.

Figure 13 presents the responses to a drop in the payments efficiency of deposits by

5%. While this shock affects the liquidity premium in a similar way as the DM preference shock shown in figure 12, it leads to an opposite effect on consumption as deposits are now less useful in the DM. Like for the DM preference shock, the central bank stabilises the increase in the liquidity premium, which leads to a larger fall in the capital premium and in deposits compared to the case without a CBDC. While these effects broadly balance each other for gross investment in the first quarters after the shock, total consumption drops less with a CBDC because the central banks' CBDC provision allows households to consume more on the DM than without a CBDC. Wit a CBDC, output increases initially but returns to steady state as investment and consumption normalise with a return of the liquidity and the capital premium to their steady state values. The existence of CBDC thus leads to a positive and less volatile output response. Likewise, with a CBDC the inflation response exhibits substantially less volatility than without.

Figure 14 shows the responses a 0.5% shock to the CBDC interest rate rule. This shock can only be analysed in the model version with CBDC and constitutes a second type of monetary policy shock, which affects the liquidity premium, in addition to the Taylor rule shock that influences the bond rate. Following the shock, the liquidity premium decreases and gradually returns to its steady state value. The reduction in the liquidity premium leads to a substitution of deposits for CBDC and enables higher consumption in the DM. The fall in the liquidity premium and deposits is accompanied by a fall in the capital premium, which on balance eases overall funding conditions and leads to an increase in investment initially, which reverses with the subsequent return of the capital and liquidity premiums to their steady state values. Overall, this induces a temporary increase in output and a decrease in inflation. Compared to the traditional monetary policy shock to the Taylor rule, the CBDC rule shock induces opposite effects on output and inflation, resulting from a larger reaction of inflation on the CM compared to the DM.

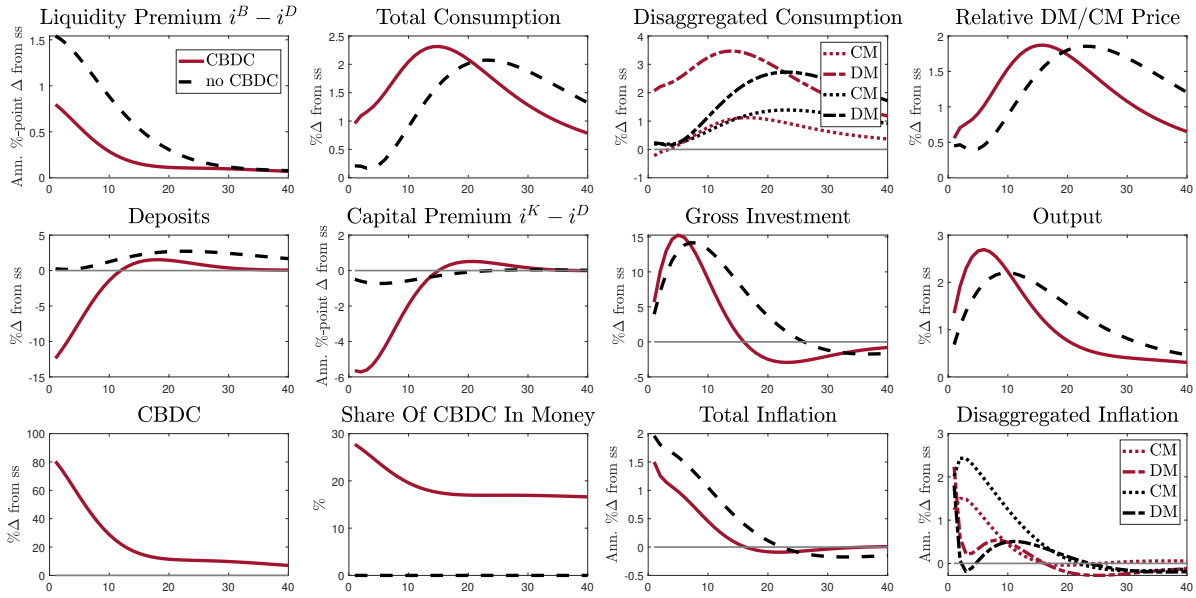

Figure 12: Responses to a 5% preference shock for DM consumption with and without CBDC

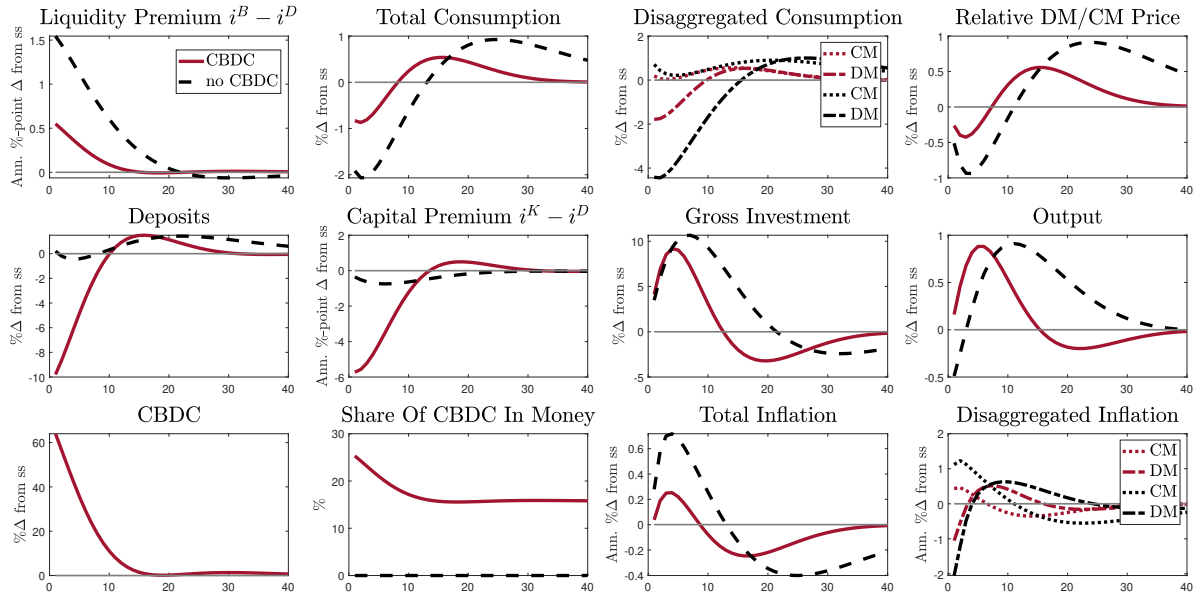

Figure 13: Responses to a 5% reduction in the payment ability of deposits with and without CBDC

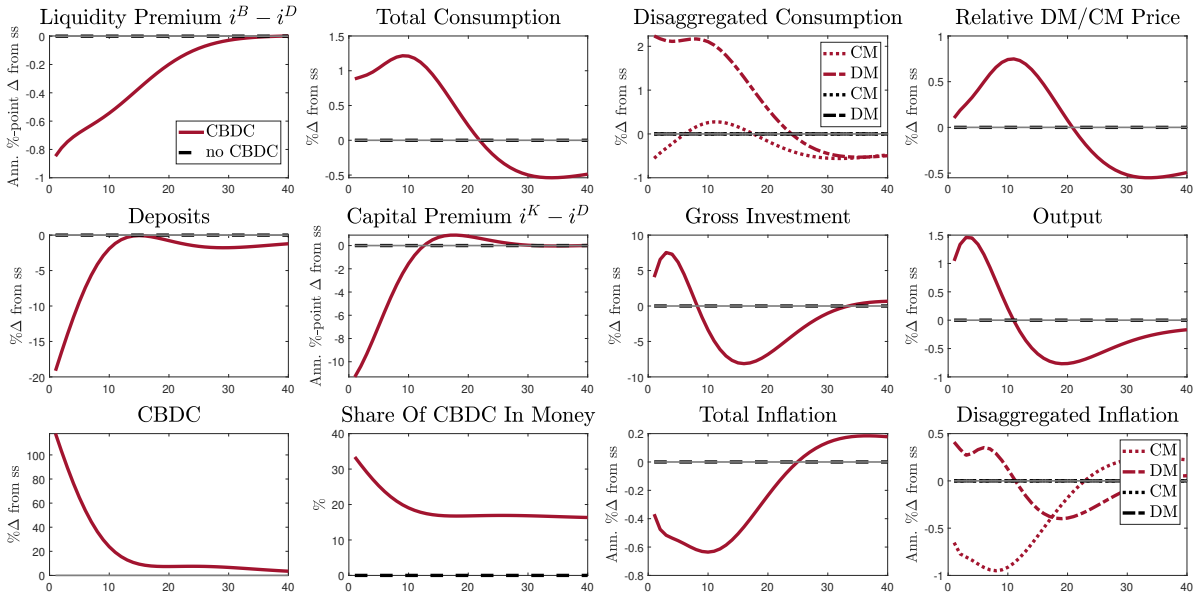

Figure 14: Responses to a 0.5% CBDC rule shock with CBDC

4 Conclusions

In this paper, we integrate a Lagos and Wright (2005)-type decentralised market into a Gertler and Karadi (2011) New-Keynesian DGSE model with financial frictions, following the approach by Aruoba and Schorfheide (2011), in order to analyse the transmission of business cycle and financial shocks as well as shocks to the means-of-exchange function of money in the presence of a CBDC. A particular advantage of this setup is that it allows us to model simultaneously the means-of-exchange and the store-of-value function of money. On the decentralised market, money is required to buy consumption goods, either with bank deposits or CBDC. On the centralised market, banks' credit provision – and with it the supply of deposits – is restricted by a leverage constraint.

Through the provision of CBDC, the central bank can affect the availability of money, thereby influencing the liquidity premium and the efficacy of exchange. At the same time, the central bank sets the interest rate on bonds according to a standard Taylor rule, which determines the opportunity cost of money and thereby affects the demand for both, deposits and CBDC. This rule defines the margin between money and bonds holdings and thus can be interpreted as targeting the store-of-value function of money. The existence of CBDC gives rise to a second policy rule that allows the central bank to target the liquidity premium and thereby affects the efficiency of exchange, i.e. the means-of-payment function of money. The interaction of the liquidity premium with money holdings and bank funding conditions gives rise to interlinkages between the centralised and the decentralised market and a rich dynamics.

We find that with a CBDC the response of output and inflation to financial shocks is dampened. Moreover, output and inflation return more quickly to their steady state values after a supply shock. In case of a demand shock, inflation responds less with a CBDC whereas output is more volatile, driven by a contemporaneous, positive response of consumption and investment as the central bank can expand the money supply without being bound by a leverage constraint. Traditional monetary policy remains effective in the presence of a CBDC, with an almost identical inflation response and a slightly less persistent output response. Like in George et al. (2020), the existence of CBDC in our model opens up a second channel for the central bank to influence the economy, which allows it to stabilise inflation more effectively than in a world in which this channel is not present.

Analysing transmission in the presence of a CBDC in a NMNK model allows for an explicit modelling of the different functions of money, which opens up new transmission channels of monetary policy through the banking sector. It would be interesting to study these new transmission channels in more detail, which would require a more elaborate modelling of the financial sector, including for instance, modelling interbank lending, central bank credit provision and collateral policies. We leave this for future research.

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A Detailed model calculation

A.1 Households:

CM maximisation problem The value function of the household in the CM is

$$V_t^{CM} = \max_{C_t^{CM}, L_t, M_t, D_t, B_t} \left\{ U(C_t^{CM}) - \chi L_t + V_t^{DM} \right\}$$

s.t. the budget constraint

$$C_t^{CM} + M_t + D_t + B_t = w_t L_t + T_t + R_t^M \hat{M}_{t-1} + R_t^D \hat{D}_{t-1} + R_t^B B_{t-1} + \Omega_t,$$

with

$$\hat{M}_t = \begin{cases} M_t - m_t^B & \text{for buyers} \\ M_t + m_t^S & \text{for sellers} \\ M_t & \text{otherwise} \end{cases} \quad \text{and} \quad \hat{D}_t = \begin{cases} D_t - d_t^B & \text{for buyers} \\ D_t + d_t^S & \text{for sellers} \\ D_t & \text{otherwise} \end{cases}$$

where m_t^B and d_t^B are the amounts of CBDC and deposits spent in the DM and m_t^S and d_t^S are CBDC and deposits received in the DM. On aggregate, sellers get what buyers pay i.e. $m_t = m_t^B = m_t^S$ and $d_t = d_t^B = d_t^S$.

The problem yields the optimality conditions:

$$V_{t,C_t^{CM}}^{CM} = U_{C_t^{CM}} - \frac{\chi}{w_t} = 0 \quad \Rightarrow \quad \varrho_t^{CM} \equiv U_{C_t^{CM}} = \frac{\chi}{w_t}$$

$$V_{t,M_t}^{CM} = -\frac{\chi}{w_t} + V_{t,M_t}^{DM} = 0, \quad V_{t,D_t}^{CM} = -\frac{\chi}{w_t} + V_{t,D_t}^{DM} = 0, \quad V_{t,B_t}^{CM} = -\frac{\chi}{w_t} + V_{t,B_t}^{DM} = 0$$

and envelope conditions

$$V_{t,\hat{M}_t}^{CM} = \varrho_{t+1}^{CM} R_{t+1}^M, \quad V_{t,\hat{D}_t}^{CM} = \varrho_{t+1}^{CM} R_{t+1}^D, \quad V_{t,B_t}^{CM} = \varrho_{t+1}^{CM} R_{t+1}^B.$$

General DM maximisation problem
Besides the quantity and price of the DM good, to complete the optimality conditions we need to define the marginal value of the assets in the DM V_{t,M_t}^{DM}, V_{t,D_t}^{DM} and V_{t,B_t}^{DM}. These can be determined by the marginal value of assets for the buyer and the seller:

$$\begin{aligned} V_t^{DM} &= \sigma V_t^{DM,B} + \sigma V_t^{DM,S} + (1-2\sigma)\beta V_{t+1}^{CM} \\ \Rightarrow V_{t,a_t}^{DM} &= \sigma V_{t,a_t}^{DM,B} + \sigma V_{t,a_t}^{DM,S} + (1-2\sigma) V_{t,a_t}^{CM} \quad \text{for} \quad a = \{M_t, D_t, B_t\} \end{aligned}$$

.

Buyer DM maximisation problem The value function of the buyer

$$\begin{aligned} V_t^{DM,B} = \max_{C_t^{DM}, m_t, d_t} & \left\{ U(C_t^{DM}) + \beta V_{t+1}^{CM}(M_t - m_t, D_t - d_t, B_t, \cdot) \right\} \\ & s.t. \quad P_t^{DM} C_t^{DM} = m_t + d_t, \\ & 0 \le m_t \le \eta_M M_t, \\ & 0 \le d_t \le \eta_D D_t \end{aligned}$$

can be reformulated using the budget constraint $m_t = P_t^{DM} C_t^{DM} - d_t$ and expressed as following Kuhn-Tucker optimization:

$$\begin{aligned}\mathcal{L}_t = &\left\{U(C_t^{DM}) + \beta V_{t+1}^{CM}\left(M_t - P_t^{DM}C_t^{DM} + d_t, D_t - d_t, B_t\right)\right\} \\ &+ \lambda_t^{m,l}\left(P_t^{DM}C_t^{DM} - d_t\right) + \lambda_t^{m,h}\left(\eta_M M_t - P_t^{DM}C_t^{DM} + d_t\right) \\ &+ \lambda_t^{d,l}\left(d_t\right) + \lambda_t^{d,h}\left(\eta_D D_t - d_t\right)\end{aligned}$$

which yields the optimality conditions:

$$\begin{aligned}FOC \quad C_t^{DM}: \quad & U_{C_t^{DM}} - \beta V_{t+1,M_t}^{CM} P_t^{DM} - P_t^{DM}(\lambda_t^{m,h} - \lambda_t^{m,l}) &= 0 \\ FOC \quad d_t: \quad & \beta V_{t+1,M_t}^{CM} - \beta V_{t+1,D_t}^{CM} + (\lambda_t^{m,h} - \lambda_t^{m,l}) - (\lambda_t^{d,h} - \lambda_t^{d,l}) &= 0\end{aligned}$$

with

$$\begin{aligned}\lambda_t^{m,l}\left(P_t^{DM}C_t^{DM} - d_t\right) = 0, \quad \lambda_t^{m,l} \geq 0 \\ \lambda_t^{m,h}\left(\eta_M M_t - P_t^{DM}C_t^{DM} + d_t\right) = 0, \quad \lambda_t^{m,h} \geq 0 \\ \lambda_t^{d,l}\left(d_t\right) = 0, \quad \lambda_t^{d,l} \geq 0 \\ \lambda_t^{d,h}\left(\eta_D D_t - d_t\right) = 0, \quad \lambda_t^{d,h} \geq 0\end{aligned}$$

From the first-order conditions for C_t^{DM}, we can derive the demand function for goods in the DM, using $V_{t,\hat{M}_t}^{CM} = \varrho_{t+1}^{CM} R_{t+1}^{M}$ from above:

$$U_{C_t^{DM}} = P_t^{DM}\left[\beta \varrho_{t+1}^{CM} R_{t+1}^{M} + (\lambda_t^{m,h} - \lambda_t^{m,l})\right]$$

From the first-order conditions for d_t we can derive the buyer's optimality conditions for transactions with of CBDC and deposits, additionally using $V_{t,\hat{D}_t}^{CM} = \varrho_{t+1}^{CM} R_{t+1}^{D}$:

$$\begin{aligned}\beta \varrho_{t+1}^{CM} R_{t+1}^{M} + (\lambda_t^{m,h} - \lambda_t^{m,l}) &= \beta \varrho_{t+1}^{CM} R_{t+1}^{D} + (\lambda_t^{d,h} - \lambda_t^{d,l}) \\ \Rightarrow (R_{t+1}^{D} - R_{t+1}^{M}) &= \frac{(\lambda_t^{m,h} - \lambda_t^{m,l}) - (\lambda_t^{d,h} - \lambda_t^{d,l})}{\beta \varrho_{t+1}^{CM}}\end{aligned}$$

Further, we get that the marginal utility of money for a buyer in the DM is value of its transactions function plus the discounted continuation value of money balances:

$$\begin{aligned}V_{t,M_t}^{DM,B} &= \beta V_{t+1,\hat{M}_t}^{CM} + \lambda_t^{m,h}\eta_M M_t \\ V_{t,D_t}^{DM,B} &= \beta V_{t+1,\hat{D}_t}^{CM} + \lambda_t^{d,h}\eta_D D_t \\ V_{t,B_t}^{DM,B} &= \beta V_{t+1,B_t}^{CM}\end{aligned}$$

Seller DM maximisation problem The sellers value function

$$V_t^{DM,S}(\cdot) = \max_{C_t^{DM}} \left\{ -\mathcal{C}(C_t^{DM}) + \beta V_{t+1}^{CM}(M_t + m_t, D_t + d_t, B_t, \cdot) \right\}$$
$$\Rightarrow V_t^{DM,S}(\cdot) = \max_{C_t^{DM}} \left\{ -\mathcal{C}(C_t^{DM}) + \beta V_{t+1}^{CM}(M_t + P_t^{DM} C_t^{DM} - d_t, D_t + d_t, B_t, \cdot) \right\}$$

yields the optimality condition:

$$FOC \quad C_t^{DM}: \quad -\mathcal{C}_{C_t^{DM}} + \beta V_{t+1,M_t}^{CM} P_t^{DM} = 0$$
$$\Rightarrow P_t^{DM} = \frac{\mathcal{C}_{C_t^{DM}}}{\beta \varrho_{t+1}^{CM} R_{t+1}^{M}}$$

which implies that the price needs to equal the marginal cost of production relative to the marginal benefit of money in the CM. This can be interpreted as the supply function of the sellers.

Further, we get that the marginal value of money for a seller in the DM is just the continuation value of the money balances he carries to the next CM:

$$V_{t,M_t}^{DM,S} = \beta V_{t+1,\hat{M}_t}^{CM}$$
$$V_{t,D_t}^{DM,S} = \beta V_{t+1,\hat{D}_t}^{CM}$$
$$V_{t,B_t}^{DM,S} = \beta V_{t+1,B_t}^{CM}$$

Equilibrium and terms of trade in the DM

Combining the demand function of the buyer $U_{C_t^{DM}} = P_t^{DM} \left[\beta \varrho_{t+1}^{CM} R_{t+1}^{M} + (\lambda_t^{m,h} - \lambda_t^{m,l}) \right]$

with the supply function of the seller $P_t^{DM} = \frac{\mathcal{C}_{C_t^{DM}}}{\beta \varrho_{t+1}^{CM} R_{t+1}^{M}}$

yields

$$U_{C_t^{DM}} = \mathcal{C}_{C_t^{DM}} \left[1 + \frac{\lambda_t^{m,h} - \lambda_t^{m,l}}{\beta \varrho_{t+1}^{CM} R_{t+1}^{M}} \right]$$

The equilibrium conditions in the DM can be divided into two regimes: An unconstrained and a constrained case. If there is enough liquidity in the DM, consumption is not constrained by money balances and one regime in which money balances limit DM consumption.

Unconstrained DM allocations

If money balances do not constrain DM consumption, the shadow price of additional liquidity is zero $\lambda_t^{m,l} = \lambda_t^{m,h} = \lambda_t^{d,l} = \lambda_t^{d,h} = 0$. In this case, equilibrium output equates

marginal utility with marginal costs of the production

$$U_{C_t^{DM*}} = \mathcal{C}_{C_t^{DM*}}$$

Thus, unconstrained DM output does not depend on deposit and CBDC balances. In this case, the is no transaction value of an additional unit of money and the value of money in the DM just reflects the continuation value of it interest payments in the CM $V_{t,a_t}^{DM} = \beta V_{t,a_t}^{CM}$ for $a = \{M_t, D_t, B_t\}$. This implies that in this case, the interest rates on CBDC, deposits and bonds are equal. This leads, together with the DM output and DM prices, to the final CM and DM optimality conditions

$$R_{t+1}^M = R_{t+1}^D = R_{t+1}^B = \frac{\varrho_t^{CM}}{\beta \varrho_{t+1}^{CM}}$$

Constrained DM allocations

Note that, as long as $M_t > 0$ and $D_t > 0$, it is not possible that both, lower and upper, constraint bind at the same time, i.e. $\lambda_t^{\cdot,h} > 0$ or $\lambda_t^{\cdot,l} > 0$. If the upper constraint is binding, the Kuhn-Tucker multiplier of the lower constraint needs to be zero and vice versa. The DM production costs $\mathcal{C}(C_t^{CM})$ are of convex nature and DM consumption utility $U(C_t^{DM})$ is of concave form. Given the DM equilibrium condition, this implies that if the buyer is constrained by its money holdings, it must be that $C_t^{DM} < C_t^{DM*}$. Therefore, if trade in the DM is constrained, it must be limited by the upper constraint, i.e. $\lambda^{m,h} \geq 0$ and $\lambda^{m,l} = 0$, as long as the DM good has a positive valuation. However, equating the Kuhn-Tucker constraints of deposits and CBDC requires that if the upper constraints on CBDC is binding, the same needs to hold true for deposits i.e. $\lambda^{d,h} \geq 0$ and $\lambda^{d,l} = 0$. This leaves us with following DM conditions that complete the DM market equilibrium

$$\begin{aligned} U_{C_t^{DM}} &= P_t^{DM}\left[\beta \varrho_{t+1}^{CM} R_{t+1}^M + \lambda_t^{m,h}\right] \\ \beta \varrho_{t+1}^{CM} R_{t+1}^M + \lambda_t^{m,h} &= \beta \varrho_{t+1}^{CM} R_{t+1}^D + \lambda_t^{d,h} \\ P_t^{DM} C_t^{DM} &= \eta_M M_t + \eta_D D_t \\ P_t^{DM} &= \frac{\mathcal{C}_{C_t^{DM}}}{\beta \varrho_{t+1}^{CM} R_{t+1}^M} \end{aligned}$$

From this we can determine DM prices $P_t^{DM} = \frac{\mathcal{C}_{C_t^{DM}}}{\beta \varrho_{t+1}^{CM} R_{t+1}^M}$, DM consumption $C_t^{DM} = \frac{\eta_M M_t + \eta_D D_t}{P_t^{CM}}$, and the shadow value of liquidity

$\lambda_t^{m,h} = \frac{\Psi_t U_{C_t^{DM}}}{P_t^{CM}} - \beta \varrho_{t+1}^{CM} R_{t+1}^M = \beta \varrho_{t+1}^{CM} R_{t+1}^M \left[\frac{\Psi_t U_{C_t^{DM}}}{\mathcal{C}_{C_t^{DM}}} - 1\right]$

and $\lambda_t^{d,h} = \beta \varrho_{t+1}^{CM} R_{t+1}^M - \beta \varrho_{t+1}^{CM} R_{t+1}^D + \lambda_t^{m,h}$.

Wie can therefore now complete the household optimality conditions for CBDC,

$$V_{t,M_t}^{DM} = \sigma V_{t,M_t}^{DM,B} + \sigma V_{t,M_t}^{DM,S} + (1-2\sigma)V_{t,M_t}^{CM} = \beta \varrho_{t+1}^{CM} R_{t+1}^{M} + \sigma \eta_M \lambda_t^{m,h} = \varrho_t^{CM}$$
$$\Rightarrow \quad 1 = \frac{\beta \varrho_{t+1}^{CM}}{\varrho_t^{CM}} R_{t+1}^{M} \left(1 + \sigma \eta_M \left[\frac{U_{C_t^{DM}}}{\mathcal{C}_{C_t^{DM}}} - 1 \right] \right)$$

deposits

$$V_{t,D_t}^{DM} = \beta \varrho_{t+1}^{CM} R_{t+1}^{D} + \sigma \eta_D \lambda_t^{d,h} = \varrho_t^{CM}$$
$$\Rightarrow \varrho_t^{CM} = \beta \varrho_{t+1}^{CM} R_{t+1}^{D} + \sigma \eta_D \left[\beta \varrho_{t+1}^{CM} R_{t+1}^{M} - \beta \varrho_{t+1}^{CM} R_{t+1}^{D} + \beta \varrho_{t+1}^{CM} R_{t+1}^{M} \left[\frac{U_{C_t^{DM}}}{\mathcal{C}_{C_t^{DM}}} - 1 \right] \right]$$
$$\Rightarrow 1 = \frac{\beta \varrho_{t+1}^{CM}}{\varrho_t^{CM}} \left[(1-\sigma\eta_D) R_{t+1}^{D} + \sigma \eta_D R_{t+1}^{M} \frac{U_{C_t^{DM}}}{\mathcal{C}_{C_t^{DM}}} \right]$$

and bonds $1 = \frac{\beta \varrho_{t+1}^{CM}}{\varrho_t^{CM}} R_{t+1}^{B}$.

A.2 Financial Intermediaries

The banker's objective is to maximise expected discounted net worth of the bank which evolves according to

$$N_{j,t+1} = R_{t+1}^{S} Q_t S_{j,t}^{B} - R_t^{D} D_{j,t+1} = (R_{t+1}^{S} - R_{t+1}^{D}) Q_t S_{j,t}^{B} + R_t^{D} N_{j,t}$$

with a balance sheet of the bank of

$$Q_t S_{j,t}^{b} = N_{j,t} + D_j.$$

The maximisation problem is

$$\begin{aligned} V_{j,t} &= \max E_t \sum_{i=0}^{\infty} (1-\theta)\theta^i \beta^{i+1} \Lambda_{t,t+1+i} N_{j,t+1+i} \\ &= \max E_t \left[(1-\theta)\beta \Lambda_{t,t+1} N_{j,t+1} + \theta \beta \Lambda_{t,t+1} V_{j,t+1} \right] = \mu_t^{s} Q_t S_{j,t}^{B} + \mu_t^{n} N_{j,t} \end{aligned}$$

where $\mu_t^s = E_t \left[(1-\theta)\beta \Lambda_{t,t+1} (R_{t+1}^{S} - R_{t+1}^{D}) + \theta \beta \Lambda_{t,t+1} \frac{Q_{j,t+1} S_{j,t+1}^{B}}{Q_{j,t} S_{j,t}^{B}} \mu_{t+1}^{s} \right]$ is the value of expanding assets by one unit financed by deposits and $\mu_t^n = E_t \left[(1-\theta)\beta \Lambda_{t,t+1} R_{t+1}^{D} + \theta \beta \Lambda_{t,t+1} \frac{N_{j,t+1}}{N_{j,t}} \mu_{t+1}^{n} \right]$ is the value of expanding net worth by one unit holding assets constant.

The optimisation of the banker is subject to the moral hazard constraint $V_{j,t} \geq \lambda Q_t S_{j,t}^{b}$. Under positive net worth, the constraint binds as long as $0 < \mu^s < \lambda$. A binding incentive

constraint restricts the leverage ratio of the bank which can be expressed as

$$\phi_t = \frac{Q_t S^B_{j,t}}{N_{j,t}} = \frac{\mu^n_t}{\lambda - \mu^s_t}.$$

The evolution of net worth and assets can also be reformulated in terms of ϕ_t and as growth rates: $\Delta^N_{t,t+q} \equiv \frac{N_{j,t+1}}{N_{j,t}} = (R^S_{t+1} - R^D_{t+1})\phi_t + R^D_{t+1}$, $\Delta^{QS}_{t,t+q} \equiv \frac{Q_{j,t+1}S^B_{j,t+1}}{Q_{j,t}S^B_{j,t}} = \frac{\phi_{t+1}N_{j,t+1}}{\phi_t N_{j,t}} = \frac{\phi_{t+1}}{\phi_t}\Delta^N_{t,t+q}$.

Banking sector aggregates are obtained by the sum of the individual banks. Aggregate bank net worth comprises the net worth of banks in operation N^E_t and newly entering banks N^N_t receiving an endowment $\omega Q_t S^B_{t-1}$:

$$N_t = N^E_t + N^N_t = \theta\left[(R^S_t - R^D_t)\phi_{t-1} + R^D_t\right] N_{t-1} + \omega Q_t S^B_{t-1}.$$

Overview of equilibrium equations

Description	**Equation**
	Households
CM consumption	$\varrho^{CM}_t = \frac{1}{C^{CM}_t} = \frac{\chi}{w_t}$
Stochastic discount factor	$\Lambda_{t,t+1} = \frac{\varrho^{CM}_{t+1}}{\varrho^{CM}_t}$
DM consumption	$C^{DM}_t = min\left[\left((\eta_m M_t + \eta_D D_t)\beta\varrho^{CM}_{t+1}R^M_{t+1}\right)^\nu, \Psi^\nu_t\right]$
CBDC demand	$1 = \frac{\beta\varrho^{CM}_{t+1}}{\varrho^{CM}_t}R^M_{t+1}\left(1 + \sigma\eta_M\left[\frac{U_{C^{DM}_t}}{\mathcal{C}_{C^{DM}_t}} - 1\right]\right)$
Deposit demand	$1 = \frac{\beta\varrho^{CM}_{t+1}}{\varrho^{CM}_t}\left[(1-\sigma\eta_D)R^D_{t+1} + \sigma\eta_D R^M_{t+1}\frac{\Psi_t U_{C^{DM}_t}}{\mathcal{C}_{C^{DM}_t}}\right]$
Bond demand	$1 = \frac{\beta\varrho^{CM}_{t+1}}{\varrho^{CM}_t}R^B_{t+1}$
	Banks
Balance Sheet	$Q_t S^B_t = N_t + D_t$
Leverage	$\phi_t = \frac{\mu^n_t}{\lambda - \mu^{qs}_t}$
Deposits	$D_t = (\phi_t - 1)N_t$
Growth rate of net worth	$\Delta^N_{t,t+1} = R^S_{t+1} + (R^K_{t+1} - R^D_{t+1})\frac{D_t}{N_t}$.
Growth rate of assets	$\Delta^S_{t,t+1} = \Delta^N_{t,t+1}\frac{\phi_{t+1}}{\phi_t}$
Value of banks assets	$\mu^S_t = E_t\left[(1-\theta)\beta\Lambda_{t,t+1}(R^S_{t+1} - R^D_{t+1}) + \theta\beta\Lambda_{t,t+1}\Delta^S_{t,t+1}\mu^s_{t+1}\right]$
Value of bank equity	$\mu^n_t = E_t\left[(1-\theta)\beta\Lambda_{t,t+1}R^D_{t+1} + \theta\beta\Lambda_{t,t+1}\Delta^N_{t,t+1}\mu^n_{t+1}\right]$

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Total net worth	$N_t = \theta \left[(R^S_{t+1} - R^D_{t+1})\phi_t + R^D_{t+1} \right] N_t + \omega Q_t S^B_{t-1}$
	Firms
CM production function	$Y^M_t = A_t \left(U_t \xi_t K_t \right)^{\alpha} L_t^{1-\alpha}$
Capital securities	$Q_t K_{t+1} = Q_t S_t$
Return on capital	$R^S_{t+1} = \left(\alpha \frac{P^M_{t+1} Y^M_{t+1}}{\xi_{t+1} K_{t+1}} + Q_{t+1} - \delta(U_{t+1}) \right) \frac{\xi_{t+1}}{Q_t}$
Depreciation function	$\delta(U_t) = \delta_c + \frac{b}{1+\psi} U_t^{1+\psi}$
Optimal capacity utilisation	$\alpha P^M_t \frac{Y^M_t}{U_t} = \delta'(U_t) \xi_t K_t$
Labour demand	$(1-\alpha) P^M_t \frac{Y^M_t}{L_t} = W_t$
Law of motion of capital	$K_{t+1} = K_t \xi_t + I^N$
Net investment	$I^n_t = I_t - \delta(U_t) \xi_t K_t$
Price of capital	$Q_t = 1 + \frac{\iota}{2} \left[\frac{I^N_t + I^{SS}}{I^N_{t-1} + I^{SS}} - 1 \right]^2 + \iota \left[\frac{I^N_t + I^{SS}}{I^N_{t-1} + I^{SS}} - 1 \right] \frac{I^N_t + I^{SS}}{I^N_{t-1} + I^{SS}}$ $- E_t \beta \Lambda_{t,t+1} \iota \left[\frac{I^N_t + I^{SS}}{I^N_{t-1} + I^{SS}} - 1 \right] \left[\frac{I^N_{t+1} + I^{SS}}{I^N_t + I^{SS}} \right]^2$
Retail output	$Y^{CM}_t = \frac{Y^M_t}{P^{disp}_t}$
Final goods price dispersion	$P^{disp}_t = \gamma P^{disp}_{t-1} (\pi^{CM}_{t-1})^{\gamma_p \epsilon} (\pi^{CM}_t)^{\epsilon} + (1-\gamma) \frac{1 - \gamma (\pi^{CM}_{t-1})^{\gamma_p (1-\gamma)} (\pi^{CM}_t)^{\gamma - 1}}{1-\gamma}^{\frac{-\epsilon}{1-\gamma}}$
Inflation	$(\pi^{CM}_t)^{1-\epsilon} = \gamma (\pi^{CM}_{t-1})^{\gamma_p (1-\epsilon)} + (1-\gamma) (\pi^{CM*}_t)^{1-\epsilon}$
Inflation optimal adjustment	$\pi^{CM*}_t = \frac{\epsilon}{\epsilon - 1} \frac{F_t}{Z_t} \pi^{CM}_t$
Optimal price adjustment (1/2)	$F_t = Y^{CM}_t P^M_t + \beta \gamma \Lambda_{t+1} (\pi^{CM}_{t+1})^{\epsilon} (\pi^{CM}_t)^{\gamma_p (1-\epsilon)} F_{t+1}$
Optimal price adjustment (2/2)	$Z_t = Y^{CM}_t + \beta \gamma \Lambda_{t+1} (\pi^{CM}_{t+1})^{\epsilon - 1} (\pi^{CM}_t)^{\gamma_p (1-\epsilon)} Z_{t+1}$
	Central Bank & Government
Taylor rule	$i_t = \left[i^{B*} + \kappa_\pi \pi_t + \kappa_y (log Y_t - log Y^*_t) \right] + \varepsilon_t$
CBDC rule	$i^M_t = i^B_t - \kappa_m \left(M_t - \overline{m} \right)$
Fisher equation bonds	$1 + i^B_{t+1} = R^B_{t+1} \frac{E_t P_{t+1}}{P_t}$
Fisher equation CBDC	$1 + i^M_{t+1} = R^M_{t+1} \frac{E_t P_{t+1}}{P_t}$
Fisher equation deposits	$1 + i^D_{t+1} = R^D_{t+1} \frac{E_t P_{t+1}}{P_t}$
CB capital purchases	$M_t = Q_t S^{CB}_t$
Central bank budget	$M_t + R^k_t S^{CB}_{t-1} = T^{CB}_t + R^M_t M_{t-1} + Q_t S^{CB}_t$
Government budget	$G + R^B_t B_{t-1} + T_t = T^{CB}_t + B_t$
Government bond issuance	$G = B_t$
	Aggregation
Total capital securities	$S_t = S^B_t + S^{CB}_t$

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CM GDP	$Y_t^{CM} = C_t^{CM} + I_t + f(\cdot)(I_t^N + I^{SS}) + G_t$
DM GDP	$Y_t^{DM} = \sigma C_t^{DM}$
DM prices	$P_t^{DM} = \frac{\mathcal{C}_{C_t^{DM}}}{\beta \varrho_{t+1}^{CM} R_{t+1}^M}$
DM inflation	$\pi_t^{DM} = \frac{P_t^{DM}}{P_{t-1}^{DM}}$
Total GDP	$Y_t^{GDP} = Y_t^{CM} + Y_t^{DM}$
Total inflation	$\pi_t = (\pi_t^{CM})^{1-s*}(\pi_t^{DM})^{s*}$

Table 2: Overview of model equations in equilibrium