Bank Market Power and Central Bank Digital Currency: Theory and Quantitative Assessment[*]

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Abstract

This paper develops a micro-founded general equilibrium model of payments to study the interaction of a central bank digital currency (CBDC) and bank market power. We find that if banks have market power in the deposit market, a CBDC can induce competition, raising the deposit rate, expands intermediation, and can lead to an increase in output. A calibration to the U.S. economy suggests that a CBDC can raise bank lending by 1.53% and output by 0.108%. We also evaluate other dimensions of the CBDC design, including acceptability, eligibility as reserves and the rule of supply, and assess the role of a CBDC as the cash usage declines.

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1 Introduction

Central banks in several countries, including China, Canada and Sweden, are considering issuing central bank digital currencies (CBDCs), a digital form of central bank money that can be used for retail payments. One frequently raised concern about a CBDC is that, since it is likely to compete with bank deposits as a payment instrument, it may increase commercial banks' funding costs and reduce bank deposits and loans, leading to bank disintermediation.[1] For example, the International Monetary Fund (IMF) staff discussion note by Mancini-Griffoli et al. (2018) argues that "as some depositors leave banks in favor of CBDC, banks could increase deposit interest rates to make them more attractive. The higher deposit rates would reduce banks' interest margins. As a result, banks would attempt to increase lending rates, though at the cost of loan demand."[2]

This study aims to formally assess this concern, *both theoretically and quantitatively*. We first develop a general equilibrium model. In the model, households allocate funds between two assets or payment instruments, cash and checkable deposits, that differ in terms of the types of exchange they can facilitate. For example, cash cannot be used in online transactions while deposits can be used via debit/credit cards or electronic transfers. Entrepreneurs have investment opportunities but no resources. Households can produce the investment good and need deposits as a means of payment. Banks act as intermediaries, creating deposits and issuing loans to entrepreneurs subject to a reserve requirement. One critical feature of our model is the imperfect competition in the deposit market (see, for example, Dreschler et al., 2017, and Wang et al., 2018, for empirical evidence).

To highlight the main mechanism of this paper, we first introduce a baseline CBDC into the model, which is a perfect substitute for bank deposits as a means of payment, bears an interest and cannot be held by banks. We examine the effect of the CBDC rate on the rates and quantities of bank deposits and loans, and on the output of the economy. We then allow the banks to hold the CBDC as reserves. Finally, we study the role of a CBDC when the cash usage in the economy continues to decline. We consider this scenario because it is of particular policy interests. It has been experienced by several countries and is cited as an

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[1]Four countries have launched a CBDC. Ecuador's *dinero electronico* failed after three years (2015-2018) of operations, largely due to the lack of trust in the government's ability to issue claims to US dollars (Ecuador has been dollarized since 2000) that it might become unable or unwilling to repay. Uruguay ran a six-month pilot with e-Pesos starting in November 2017. In December 2019, the Central Bank of Bahamas started its CBDC pilot Project Sand Dollar. In these three countries, bank disintemediation is less of a concern; in fact, a CBDC is cited as a means to improve financial inclusion and access to financial services. In April 2020, China's central bank launched a pilot program of its Digital Currency Electronic Payment (DCEP) in four cities.

[2]See also the BIS report by the Committee on Payments and Market Infrastructures (2018).

important reason for issuing a CBDC. The COVID-19 pandemic may further accelerate this trend.[3] We also discuss other design choices in terms of acceptability and the rule of supply (for example, fixed quantity or rate).

Our model predicts that if banks have market power in the deposit market, the impact of a CBDC is *non-monotone* in its interest rate. It expands bank intermediation if its rate is in some intermediate range and causes disintermediation if its rate is too high. This finding is robust to how we model the deposit market and the loan market. In the main text, we focus on Cournot quanity competition in the deposit market and perfect competition in the loan market. In the Appendix, we allow for an imperfectly competitive loan market, and also study a model with price competition in the deposit market following Burdett and Judd (1983) and Allen et. al (2012).[4]

The main mechanism through which a CBDC "crowds in" bank intermediation works as follows. In an imperfectly competitive deposit market, banks restrain deposit supply to keep interest rates on deposits lower than (or equivalently, the price of deposits higher than) the level under perfect competition. A CBDC offers an outside option to depositors and sets an interest rate floor for bank deposits.[5] This interest floor reduces commerical banks' incentive to restrain deposit supply, because it limits the reduction in the deposit rate. As a result commerical banks supply more deposits, lower the loan rate and expand lending when the reserve requirement is binding.

Interestingly, the CBDC may or may not be used in the equilibrium depending on its interest rate. But it can have a positive effect on deposits, loans and output even if it has zero market share. The existence of a CBDC as an outside option forces banks to match the CBDC rate and create more deposits.[6] This is different from the standard competition effects where total quantity increases as a result of more suppliers. The CBDC can induce existing banks

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[3]For example, see the Payment Canada report "COVID-19 pandemic dramatically shifts Canadians' spending habits."

[4]We thank Eric Smith for suggesting this version of the model.

[5]This suggests that the CBDC rate could have stronger pass-through to deposit rate than traditional monetary policy tools, which do not seem to affect deposit rates much. For example, the policy rate in the U.S. increased by 2% from 2016 to 2018, but commercial banks barely moved their deposit rates, measured as the national rate on non-jumbo deposits (less than $100,000) for money market, savings, or interest checking accounts (source: FDIC, Money Market [MMNRNJ], Savings [SAVNRNJ] and Interest Checking [ICNRNJ] data series retrieved from FRED, April 15, 2020). On a related note, Berentsen and Schar (2018) argue that a CBDC would make monetary policies more transparent as the interest rate on a (publicly accessible) CBDC would set a floor for the deposit rate.

[6]This insight is closely related to Lagos and Zhang (2018), who show that monetary policies discipline the equilibrium outcome by setting the value of the outside option and can be effective even if money is not used in equilibrium.

to supply more. A policy implication is that one should assess the effectiveness of a CBDC based on its effect on deposits or the deposit rate instead of its usage.

Calibrating our model to the U.S. economy, we find that the baseline CBDC expands bank intermediation if the interest rate on the CBDC is between 0.3049% and 1.28%. At the maximum, a CBDC can increase loans and deposits by about 1.53% and the total output by about 0.108%. The positive effect reverses if the CBDC rate exceeds 1.18%. To stay break-even, banks are forced to raise the lending rate to compensate for the interest paid on deposits, which reduces the equilibrium quantity of loans and deposits.[7] If the CBDC serves as reserves, it promotes bank intermediation for a wider range of interest rates and its effect is also slightly stronger. Finally, even if a CBDC bears zero interest, it can restrict banks' market power and improve intermediation as the economy becomes increasingly cashless. Without a CBDC, banks could reduce intermediation and pay negative deposit rates.

Our study is closely related to two concurrent papers. Keister and Sanches (2019) focus on the welfare implication of an interest-bearing CBDC when the banking sector (modelled as a bank-firm combination) is perfectly competitive. Banks are subject to financial frictions because of the limited pledgeability of their projects. Keister and Sanches (2019) highlight a trade-off of the CBDC: it always crowds out bank intermediation, but can promote efficiency in exchange. The benefit of efficient exchange dominates the cost associated from disintermediation if financial frictions are not very big.

Andolfatto (2018) studies the effect of a CBDC on bank intermediation in an economy with heterogeneous households and a monopolistic bank. He uses the overlapping generations (OG) framework where young households save for old age in cash, deposits or a CBDC. The latter two require costly access to the banking system, so poor households save only in cash. He shows that a CBDC could compel the bank to increase the deposit rate, which increases financial inclusion and bank deposits. On the lending side, he assumes that the central bank lends unlimitedly to the commercial bank at a fixed rate, which fully determines the level of loans and disconnects the bank's loans from its deposits.

Compared to these two papers, our framework is more suitable for quantifying the effect of a CBDC and accommodates various design choices as the payment landscape evolves. First, our model captures a full spectrum of competitiveness. If the number of banks is one, the

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[7]As suggested by Meaning et al. (2018), an important research question regarding CBDC is "... at which point do the benefits of a new competitive force for the banking sector get outweighed by the negative consequences of the central bank disintermediating a large part of banks business models?" Our calibration exercise allows us to calculate the value of the CBDC rate at which the effect of a CBDC on bank intermediation reverses from positive to negative.

banking sector is monopolistic as in Andolfatto (2018). If this number tends to infinity, the banking sector is perfectly competitive as in Keister and Sanches (2019). We use data to discipline the level of competitiveness, which is crucial for quantifying the impact of a CBDC. Second, we explicitly model cash, deposits and a CBDC as different but imperfectly substitutable payment instruments to facilitate different types of transactions. This allows us to discuss the design of the CBDC in terms of its acceptability and its effect when the payment landscape evolves, e.g., the cash usage declines. Finally, modelling the reserve requirement allows us to discuss another issue regarding the design of a CBDC: whether it can be held as bank reserves.

This paper focuses on the effects of a CBDC on competition and abstracts from issues such as banks' risk taking behavior and financial stability. For work on these aspects, see Monnet et. al (2019), Chiu et. al (2019) and Fernandez-Villaverde et al. (2020a,b).

There are a number of papers that study other implications of a CBDC. Barrdear and Kumhof (2016) develop a DSGE model and estimate that issuing a CBDC could increase GDP by up to 3% through reducing real interest rates. Brunnermeier and Niepelt (2019) derive conditions under which the issuance of inside money and outside money are equivalent, even if inside money and outside money have liquidity or return differences. Their results imply that introducing a CBDC does not necessarily change macroeconomic outcomes. Davoodalhosseini (2018) studies a model where a CBDC allows balance-contingent transfers, but is more costly to use than cash. Then the co-existence of cash and the CBDC may not be optimal, because cash can serve as an outside option for agents, restricting the central bank's power in implementing balance-contingent transfers. Williamson (2019) argues that a CBDC can raise issues regarding independence of the central bank and scarcity of assets eligible to back the CBDC. Dong and Xiao (2019) show that certain types of CBDC can be useful in implementing a negative policy rate.[8]

Our paper is related to the literature on transmission channels of monetary policy through the banking system. In their seminal work, Bernanke and Blinder (1992) propose a bank reserve channel. A higher interest rate increases the opportunity cost of holding reserves, leading banks to reduce their lending. This channel also operates in our model. Dreschler, Savov, and Schnabl (2017) propose a transmission channel based on banks' market power in deposit markets. A higher nominal interest rate makes cash more expensive relative to deposits, and banks with market power raise the spread between the nominal interest rate

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[8]For policy discussions on CBDC, see Agur et al. (2019); Mancini-Griffoli et al. (2018); Chapman and Wilkins (2019); Davoodalhosseini and Rivadenyra (2018); Davoodalhosseini et al. (2018); Engert and Fung (2017); Fung and Halaburda (2016); and Kahn et al. (2018).

and the deposit rate. The mechanism through which a higher interest on a CBDC raises the deposit rate and quantity in our model is similar to the mechanism through which an expansionary policy works in their paper: both policies reduce the banks' market power in the deposit market. Different from their paper, we model lending and the payments arrangements explicitly. These features allow us to study the macroeconomic effects on production and consumption, and to conduct various counterfactual analyses regarding the effects of CBDCs with different design choices.[9]

Finally, our paper contributes to the New Monetarist literature with financial intermediation by modeling imperfect competition in inside money creation. Berentsen, Camera, and Waller (2008) first incorporate banking into the Lagos and Wright (2005) model. Their banking sector is perfectly competitive and does not create inside money. Gu et al. (2018) show that banking sector is inherently unstable. Consistent with their findings, our banking sector is potentially unstable because it may lead to multiple equilibria; see Appendix B. Dong et al. (2016) study a model of oligopolistic banks that face mismatch in the timing of payments, and show that both bank profits and welfare are non-monotone in the number of banks.

This paper is organized as follows. Section 2 introduces the baseline model, where there is no CBDC. Section 3 derives the equilibrium of the baseline model. Section 4 studies the impact of a CBDC that cannot be held by commercial banks to illustrate the main mechanism of this paper. Section 5 analyzes the case where commercial banks can hold CBDC as reserves. Section 6 calibrates the model and assesses the quantitative implications. Section 7 discusses the robustness of our results in different extensions of the model. It also discusses two alternative designs of a CBDC. Section 8 concludes. The omitted proofs and calculations are collected in Appendix A. Extensions and further discussions come in other appendices.

2   Environment

Time is discrete and continues forever from 0 to infinity. There are four types of agents: a continuum of households with measure 2, a continuum of entrepreneurs with measure 1, a finite number of N bankers, and the government. The discount factor from current to the

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[9]Using a variation of Dreschler et al. (2017) model, Kurlat (2018) shows that banks' market power raises the cost of inflation. Scharfstein and Sunderam (2016) propose another transmission channel based on banks' market power in *loan* markets. As the nominal interest rate increases, the banks reduce their markup due to lower demand for loans. Quantitatively, Wang et. al (2019) estimate a structural banking model and show that the effect of banks' market power in monetary policy transmission is sizable and comparable to that of bank capital regulations. More specifically, when the interest rate is low, the deposit channel is more important. Given that we focus on environments with low interest rates in our counterfactual exercise, their finding supports our assumption of banks' market power in the deposit market.

next period is $0 < \beta < 1$. In each period t, agents interact sequentially in two stages: a frictionless centralized market (CM), and a frictional decentralized market (DM). There are two perishable goods: good x in the CM and good y in the DM.

Households are divided into two permanent types, buyers and sellers, each with measure 1. In the CM, both types work and consume x. Their labor h is transformed into x one-for-one. In the DM, buyers and sellers meet bilaterally and trade good y. Buyers want to consume y, which can be produced on the spot by the sellers. The utility from consumption is $u(y)$ with $u' > 0$, $u'(0) = \infty$, and $u'' < 0$. The disutility from production is normalized to y. Let y^* be the socially efficient DM consumption, which solves $u'(y^*) = 1$. To summarize, buyers and sellers have period utilities given respectively by

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\begin{aligned}
\boldsymbol{U}^B(x, y, h) &= U(x) - h + u(y), \\
\boldsymbol{U}^S(x, y, h) &= U(x) - h - y.
\end{aligned}
$$

Young entrepreneurs are born in the CM and become old and die in the next CM. Enterpreneurs cannot work in the CM and care about only consumption when old. Young enterpreneurs are endowed with an investment opportunity that transforms x current CM goods to $f(x)$ CM goods in the next period, where $f'(0) = \infty$, $f'(\infty) = 0$, $f' > 0$, and $f'' < 0$.

Given the preferences and endowment, there are gains from trade between buyers and sellers, and between entrepreneurs and households. Buyers would like to consume DM goods produced by sellers, and entrepreneurs would like to borrow from houeholds to invest in their investment opportunities. However, entrepreneurs lack commitment and households cannot enforce debt repayment, so credit arrangement among them is not viable.

Like entrepreneurs, young bankers are born in the CM and will become old and die in the next CM.[10] They cannot work in the CM and consume only when old. Unlike households and entrepreneurs, bankers can commit to repay and enforce payment (refer to Gu et al., 2018, for the discussion of the endogenous emergence of banks). Each of the bankers runs a bank. As a result, banks can act as intermediaries between households and entrepreneurs to finance the investment projects. A bank has the option to issue illiquid or liquid deposits. Illiquid (time) deposits cannot be used to make payments, while liquid (checkable) deposits can.

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[10]Infinitely-lived banks complicate expositions but have little impact on the results. Banks do not have incentives to retain profits for investment because deposit financing is cheaper.

The government issues fiat money, which is a physical token and can be used as a means of payment. Throughout the paper, we use "fiat money" and "cash" interchangeably. The supply of fiat money M_t grows at a constant gross rate $\mu > \beta$. The change in the supply is implemented as lump-sum transfers to (if $\mu > 1$) or taxes on (if $\mu < 1$) households. The government also stipulates a reserve requirement that a bank's cash holding must cover at least χ fraction of its checkable deposits.

As in Zhu and Hendry (2019), there are three types of meetings in the DM, depending on which of the two means of payment, cash and deposits, can be used for transactions. From a buyer's perspective, with $\alpha_1 > 0$ probability, a buyer enters into a type 1 meeting, where only fiat money can be used. With $\alpha_2 > 0$ probability, a buyer enters into a type 2 meeting, where only bank deposits can be used. With $\alpha_3 \geq 0$ probability, a buyer enters into a type 3 meeting, where both can be used. The three types of meetings can be interpreted as follows. Type 1 meetings are transactions in local stores that do not have access to debit cards; type 2 meetings are online transactions where the buyers and sellers are spatially separated and can only use debit cards or bank transfers; and type 3 meetings occur at local stores with point-of-sale (POS) machines, and hence both payment methods are accepted.

Agents in our model engage in the following activities. In every CM, young bankers issue deposits for two purposes. First, banks issue deposits to households in exchange for fiat money which can be kept as bank reserves. Second, banks offer loans to entrepreneurs in the form of deposits, which entrepreneurs use to buy x from households for investment. In the DM, buyers use a combination of cash and checkable deposits to purchase goods y from sellers. In the following CM, deposits and loans are settled. Entrepreneurs sell some of the investment output for cash or deposits to settle bank loans and retain the remaining output for their own consumption. Having collected the loan repayments, bankers redeem the deposits held by the households and retain the remaining profit for their own consumption. Figure 1 presents the timeline for all agents.

For the analysis in the main text, we assume that banks engage in Cournot competition in the deposit market, but the lending market is perfectly competitive. We choose to focus on this structure because there is stronger evidence of imperfect competition in the deposit market than in the loan market (Dreschler et al., 2017; Wang et al., 2018). In Appendix C, we extend the model to the case where the lending market also features imperfect competition.
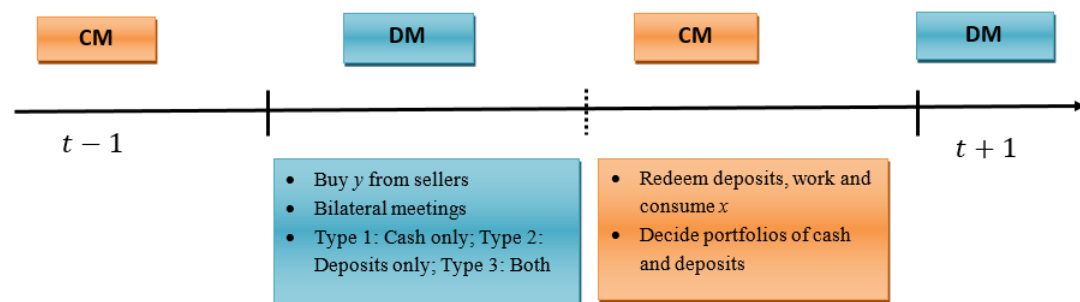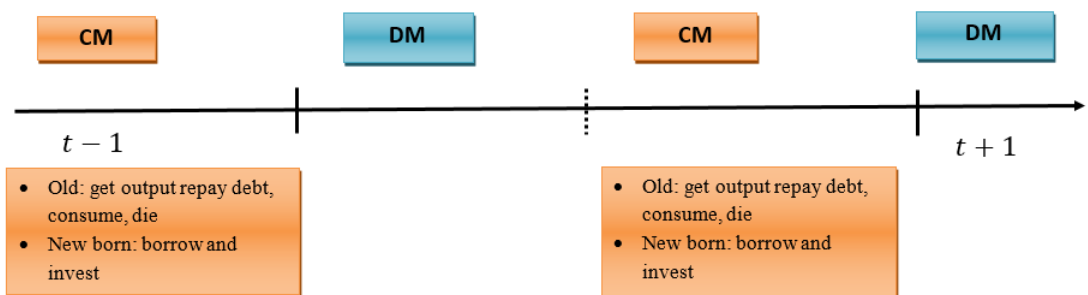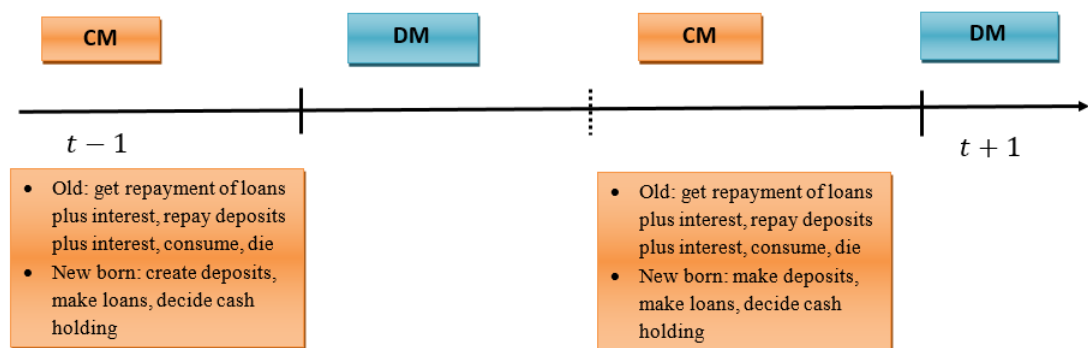
(a) Buyers
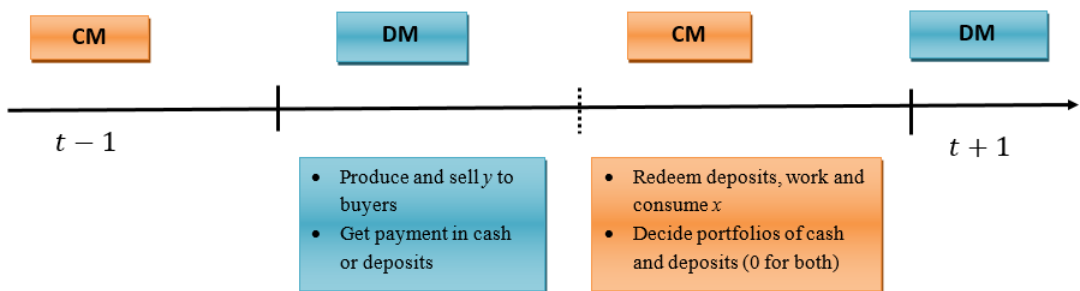
(b) Sellers

(c) Entrepreneurs

(d) Bankers

Figure 1: Timeline

3 Equilibrium without a CBDC

In this paper, we focus on the steady-state monetary equilibrium, where cash has positive value. It takes four steps to solve for the equilibrium. First, we characterize the household's problem to derive the demand for cash and bank deposits. Second, we lay out the problem faced by the bank, incorporating the household demand for deposits, to derive the aggregate supply curve for loans. Third, we derive the demand curve for loans. Finally, we equate the supply and demand for loans to derive the market clearing loan rate, and combine it with the solutions to all agents' problems to characterize the equilibrium deposit rate, real cash balances (held by households and banks as reserves), and the quantities of deposits and loans.

3.1 Households

We first examine the buyer's maximization problem, and then the seller's problem. Let W and V be the value functions of households in the CM and DM, respectively. In the following, we suppress the time subscript and use prime to denote variables in the next period.

In the CM, a buyer chooses consumption x, labor h, and the real cash, checkable and time deposit balances in the next period, z', d' and b'.[11] The buyer's value function is

$$W^B(z,d,b) = \max_{x,h,z',d',b'} \left\{ U(x) - h + \beta V^B(z',d',b') \right\}$$

$$\text{st. } x = h + z + d + b + T - \frac{\phi}{\phi'}z' - \psi d' - \psi_b b',$$

where ϕ is the value of cash in terms of CM good, and $\psi d'$ and $\psi_b b'$ are the real value of checkable and time deposits today, respectively. The real return on cash balances is $\phi'/\phi - 1$, the real interest rate on checkable deposits is $1/\psi - 1$, and the real interest rate on time deposits is $1/\psi_b - 1$. Substitute out h using the budget equation and rewrite the buyer's CM problem as

$$W^B(z,d,b) = z + d + b + T + \max_x \left[U(x) - x \right]$$

$$+ \max_{z',d',b'} \left\{ -\frac{\phi}{\phi'}z' - \psi d' - \psi_b b' + \beta V^B(z',d',b') \right\}.$$

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[11]The type of the DM meeting is not revealed until the start of the DM. Therefore, buyers hold a portfolio of cash and bank deposits.

Note that $W^B(z, d, b)$ is linear in z, d, and b. The first-order conditions (FOCs) are

$$x \; : \; U'(x) = 1,$$
$$z' \; : \; \frac{\phi}{\phi'} \geq \beta V_1^B(z', d', b'), \; \text{with equality if } z' > 0,$$
$$d' \; : \; \psi \geq \beta V_2^B(z', d', b'), \; \text{with equality if } d' > 0,$$
$$b' \; : \; \psi_b \geq \beta V_3^B(z', d', b'), \; \text{with equality if } b' > 0,$$

where the subscripts indicate the derivative with respect to corresponding arguments. Two standard results are that all buyers choose the same portfolio (z', d', b'), and $W^B(z, d, b)$ is linear in (z, d, b) with $W_1^B(z, d, b) = W_2^B(z, d, b) = W_3^B(z, d, b) = 1$.

The buyer's DM problem is

$$V^B(z, d, b) = \alpha_1 [u \circ Y(z) - P(z)] + \alpha_2 [u \circ Y(d) - P(d)] \tag{1}$$
$$+ \alpha_3 [u \circ Y(z + d) - P(z + d)] + W^B(z, d, b),$$

where $Y(\cdot)$ and $P(\cdot)$ are the terms of trade (TOT) and represent the amount of good y being traded and the amount of payment, respectively. The TOT depends on the buyer's usable liquidity, which varies according to the type of meetings. We will discuss the TOT in detail later.

Now we characterize the seller's problem. The seller enters the DM with zero liquidity balances, or $z' = d' = 0$, because he/she does not need liqiuidity in the DM. Using this result, we can formulate the seller's CM problem as

$$W^S(z, d, b) = \max_{x, h} \left\{ U(x) - h + \beta V^S(0, 0, b') \right\}$$
$$\text{st. } x + \psi_b b' = h + z + d + b + T.$$

Similar to the buyer's problem, $U'(x) = 1$, and W^S is linear in z, d and b. The seller's DM problem is

$$V^S(0, 0, b) = \alpha_1 [P(\tilde{z}) - Y(\tilde{z})] + \alpha_2 \left[P(\tilde{d}) - Y(\tilde{d}) \right]$$
$$+ \alpha_3 \left[P(\tilde{z} + \tilde{d}) - Y(\tilde{z} + \tilde{d}) \right] + W^S(0, 0, b),$$

where $\tilde{d}$ and $\tilde{z}$ are the cash and deposit holdings of his/her trading partner.

The TOT are determined by buyers making take-it-or-leave-it offers. Let $\mathcal{L}$ be the buyer's

total available liquidity, which is equal to z in type 1 meetings, d in type 2 meetings, and $d + z$ in type 3 meetings. The buyer offers output-payment pair (y, p) to solve

$$\max_{y,p} [u(y) - p] \text{ s.t. } p \geq y \text{ and } p \leq \mathcal{L},$$

where the first constraint is the seller's participation constraint and the second the liquidity constraint. The TOT as a function of the buyer's total available liquidity $\mathcal{L}$ are

$$Y(\mathcal{L}) = P(\mathcal{L}) = \min(y^*, \mathcal{L}). \tag{2}$$

In words, if the buyer has enough payment balances to purchase the optimal amount, then the optimal amount is traded; otherwise, the buyer spends all available payment balances.

Combining the FOCs of buyers with respect to z' and d' in the CM and equations (1) and (2), we obtain the buyer's demand for payment balances,

$$\frac{\phi}{\beta \phi'} = \alpha_1 \lambda(z') + \alpha_3 \lambda(z' + d') + 1,$$
$$\frac{\psi}{\beta} = \alpha_2 \lambda(d') + \alpha_3 \lambda(z' + d') + 1,$$

where $\lambda(\mathcal{L}) = \max[u'(\mathcal{L}) - 1, 0]$ is the liquidity premium. Note that the demand for cash and deposits is positive if $u'(0) = \infty, \alpha_1 > 0$ and $\alpha_2 > 0$. At the steady state, z and d are constant over time. Then $\phi/\phi' = \mu$ and the demand for liquid balances (z, d) is given by

$$\iota = \alpha_1 \lambda(z) + \alpha_3 \lambda(z + d), \tag{3}$$
$$\frac{\psi}{\beta} - 1 = \alpha_2 \lambda(d) + \alpha_3 \lambda(z + d), \tag{4}$$

where $\iota = \mu/\beta - 1$ is the nominal interest rate using the Fisher's equation. These two equations are intuitive. The first one states that the marginal cost of holding cash (the left-hand side) should be equal to its marginal benefit (the right-hand side). The latter comes from the fact that the buyer can use the marginal unit of cash in type 1 and type 3 meetings to derive $\lambda(z)$ and $\lambda(z + d)$ additional units of utility, respectively, from consumption. The second equation is for checkable deposits and has a similar interpretation.

Here, (3) defines the aggregate demand for cash balances z as a function of d. Given this, (4) defines the aggregate inverse demand function for checkable deposits, $\psi = \Psi(d)$. It has the following properties: $\Psi(0) = \infty$, $\Psi(d) = \beta$ for $d \geq y^*$, $\Psi'(d) < 0$ for $d < y^*$, and $\Psi'(d) = 0$

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for $d \geq y^*$.[12]

Finally, the demand for time deposits is separate from the demand for liquid assets and is given by

$$\psi_b \;=\; \beta.$$

In words, since time deposits have no liquidity value, the rate of return of time deposits must compensate for discounting across time.

3.2 Banks

Banks issue two types of deposits, checkable deposits (d) and time deposits (b), to households, and invest in two assets, cash (z) and loans (ℓ).[13] The N bankers engage in Cournot competition in the deposit market and are fully competitive in the loan market (where they take the loan rate, ρ, as given). Bankers face a reserve requirement.[14] At the end of each CM, the real value of a banker's cash holding must be at least χ fraction of the total checkable deposits, where χ is a policy parameter set by the government.

Formally, banker j solves the following maximization problem, taking the price of time deposits ($\psi_b = \beta$), the market loan rate (ρ) and other banks' checkable deposit quantities ($D_{-j} = \sum_{i \neq j} d_i$) as given:[15]

$$\max_{z_j, \ell_j, d_j, b_j} \left\{ (1 + \rho)\,\ell_j + \frac{z_j}{\mu} - d_j - b_j \right\} \tag{5}$$
$$\text{s.t.} \quad \ell_j + z_j = \Psi\left(D_{-j} + d_j\right) d_j + \beta b_j,$$
$$z_j \geq \chi \Psi\left(D_{-j} + d_j\right) d_j.$$

The banker maximizes consumption in the second period of life. He/She receives the repayment of loans (principal plus interest) from the entrepreneurs, $(1 + \rho)\,\ell_j$, the inflation-adjusted value of money holdings, and redeems the deposits d_j and b_j. Here z_j is real cash balances in the banker's first CM. These cash balances are worth $\phi' z_j / \phi = z_j / \mu$ in the sec-

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[12]From (3) and (4), $\Psi'(d) = \alpha_2 \beta \lambda'(d) + \frac{\alpha_1 \alpha_3 \beta \lambda'(z+d)\lambda'(z)}{\alpha_1 \lambda'(z) + \alpha_3 \lambda'(z+d)} \leq 0$, with strict inequality if $d < y^*$.

[13]In most cases, the time deposit is not issued in the equilibrium. However, it eliminates some uninteresting technical issues. Our main results remain unchanged if we remove it.

[14]For simplicity, we consider cash reserves in the benchmark model. As discussed in Section 7, allowing interest-bearing reserves does not change the main findings of the model.

[15]Note that banks cannot affect the price of time deposits, which is fixed at $\psi_b = \beta$ from the household's problem. To ease notation, we suppress the dependence of Ψ on ι.

ond CM because of inflation. And d_j and b_j are the after-interest values of deposits in the banker's second CM. The before-interest values of deposits are $\Psi(D_{-j}+d_j)d_j$ and βb_j, which capture the amount of resource banker j has in his first CM. The maximization problem has two constraints. The first constraint is the balance sheet identity of the bank at the end of the banker's first CM. The right-hand side is the liability, the before-interest real value of checkable and time deposits. The left-hand side is the asset, which includes cash and loans. The second constraint reflects the reserve requirement. We also implicitly impose that d_j, b_j, z_j, ℓ_j are non-negative throughout the paper.

In the following, we analyze the Cournot competition in the deposit market given the loan rate ρ. We focus on the symmetric equilibrium where every bank makes the same choice. Note that if $1+\rho > 1/\beta$, then the bank can make unlimited profits by issuing time deposits and investing in loans. As a result, $1+\rho \leq 1/\beta$ in equilibrium. There are four cases depending on the magnitude of $1+\rho$ relative to the gross return on cash, $1/\mu$, and the gross return on time deposits, $1/\beta$.

Case 1: cash has a higher return than loans. If $1+\rho < 1/\mu$, then the bank does not invest in loans ($\ell = 0$) because its return is dominated by cash. The bank's asset side consists of only cash, and the reserve requirement does not bind. The bank's problem can be rewritten as

$$\max_{z_j,d_j,b_j} \left\{ \frac{z_j}{\mu} - d_j - b_j \right\}$$
$$\text{s.t. } z_j = \Psi(D_{-j}+d_j)d_j + \beta b_j.$$

Use the balance sheet identity to eliminate z_j and obtain

$$\max_{d_j,b_j} \left\{ \frac{\Psi(D_{-j}+d_j)d_j + \beta b_j}{\mu} - d_j - b_j \right\}$$

The first-order condition for d_j is

$$d_j \quad : \quad \Psi'(D_{-j}+d_j)d_j + \Psi(D_{-j}+d_j) = \mu.$$

The derivative with respect to b_j is $\beta/\mu - 1 < 0$, implying $b_j = 0$; i.e., the bank does not issue time deposits because the return on cash is less than the deposit rate required by the household, $1/\beta$. In a symmetric equilibrium, each bank issues d_μ units of checkable deposits

and invests only in cash ($z = \Psi(Nd_\mu)d_\mu$), where d_μ solves

$$\Psi'(Nd_\mu)d_\mu + \Psi(Nd_\mu) = \mu.$$

Case 2: cash and loans have the same return. If $1 + \rho = 1/\mu$, then the bank is indifferent between investing in loans and cash reserves as long as the reserve requirement is satisfied. The supply of (checkable and time) deposits remains the same as in case 1. The supply of loans for each bank lies on the interval $[0, (1 - \chi)\Psi(Nd_\mu)d_\mu]$.

Case 3: loans have a higher return than cash. If $1 + \rho > 1/\mu$, then the reserve requirement binds, and we can rewrite the bank's problem using the constraints to eliminate ℓ_j and z_j in the objective function in (5) as

$$\max_{d_j, b_j} \left\{ (1 + \rho)[(1 - \chi)\Psi(d_{-j} + d_j)d_j + \beta b_j] + \frac{\chi\Psi(d_{-j} + d_j)d}{\mu} - d_j - b_j \right\}.$$

The first-order condition for d_j is

$$d_j \quad : \quad \Psi'(d_{-j} + d_j)d_j + \Psi(d_{-j} + d_j) = \frac{1}{(1 + \rho)(1 - \chi) + \chi/\mu}.$$

In a symmetric equilibrium, the supply of checkable deposits for each bank, d, solves

$$\Psi'(Nd)d + \Psi(Nd) = \frac{1}{(1 + \rho)(1 - \chi) + \chi/\mu}, \tag{6}$$

where the denominator is the gross rate of return on the bank's assets, which is a weighted average of the gross returns on loans and on cash. In terms of time deposits, the first-order derivative with respect to b_j is $(1 + \rho)\beta - 1$. We can further divide case 3 into two sub-cases depending on the relative magnitudes of $1 + \rho$ and β.

Case 3a. If $1/\mu < 1 + \rho < 1/\beta$, then banks will not issue time deposits because the required return on time deposits exceeds the return on loans. The bank splits its checkable deposits between cash reserves in the amount of $z = \chi\Psi(Nd)d$ and loans in the amount of $\ell = (1 - \chi)\Psi(Nd)d$.

Case 3b. If $1 + \rho = 1/\beta$, then banks will start issuing time deposits. The amount of checkable deposits is given by d_β, which solves equation (6) at $\rho = 1/\beta - 1$:

$$\Psi'(Nd_\beta)d_\beta + \Psi(Nd_\beta) = \frac{1}{(1 - \chi)/\beta + \chi/\mu}.$$

The amount of cash reserves is $z_\beta = \chi\Psi(Nd_\beta)d_\beta$. The amount of loans supplied by each bank is $\ell_\beta = [(1-\chi)\Psi(Nd_\beta)d_\beta, \infty)$. To finance the loans, the bank uses both checkable deposits d_β and time deposits $b = [\ell_\beta - (1-\chi)\Psi(Nd_\beta)d_\beta]/\beta$.

Notice that the loan supply may be indeterminant for certain values of ρ. Therefore, we say that the Cournot game has a unique symmetric equilibrium if the checkable deposit supply is unique. To establish existence and uniqueness of the equilibrium in the Cournot game, the following assumption is imposed throughout the paper.

Assumption 1 *a) Given any $D_{-j} \in [0, y^*)$ and $\kappa > \beta$, either there exists a unique $d_j > 0$ such that $\Psi'(D_{-j} + d)d + \Psi(D_{-j} + d) \gtrless \kappa$ if $d \lessgtr d_j$, or $\Psi'(D_{-j} + d)d + \Psi(D_{-j} + d) < \kappa$ for all $d \geq 0$. b) $\Psi'(Nd)d + \Psi(Nd)$ decreases with d on $[0, y^*/N)$ and is larger than μ for sufficiently small d.*

Part (a) of this assumption states that $\Psi'(D_{-j} + d)d + \Psi(D_{-j} + d) - \kappa$ as a function of d crosses the horizontal axis from the above and at most once. It guarantees that the best response of banker j to any amount of checkable deposits (less than y^*) created by other banks is unique. Part (b) plays three roles. First, it ensures the uniqueness of the symmetric Nash equilibrium in the Cournot game. Second, it implies that the equilibrium checkable deposits is increasing in ρ. Lastly, it guarantees that banks issue checkable deposits for any ρ.[16]

Proposition 1 *The Cournot game has a unique symmetric pure strategy equilibrium if $\rho \leq 1/\beta - 1$. In the equilibrium, each bank supplies $d(\rho)$ checkable deposits, where $d(\rho)$ is increasing in ρ and solves:*

$$\Psi'(Nd)d + \Psi(Nd) = \min\left\{\mu, \frac{1}{(1+\rho)(1-\chi) + \chi/\mu}\right\}. \tag{7}$$

Proof. *See Appendix A.1.* ∎

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[16]One can show that Assumption 1 holds if $u(y) = \frac{y^{1-\sigma}}{1-\sigma}$, with $\sigma < 1$ and $\alpha_3 = 0$. By continuity, this would hold if α_3 is sufficiently small. The last part of Assumption 1(b) is introduced only to simplify presentation and is not crucial for our main results.
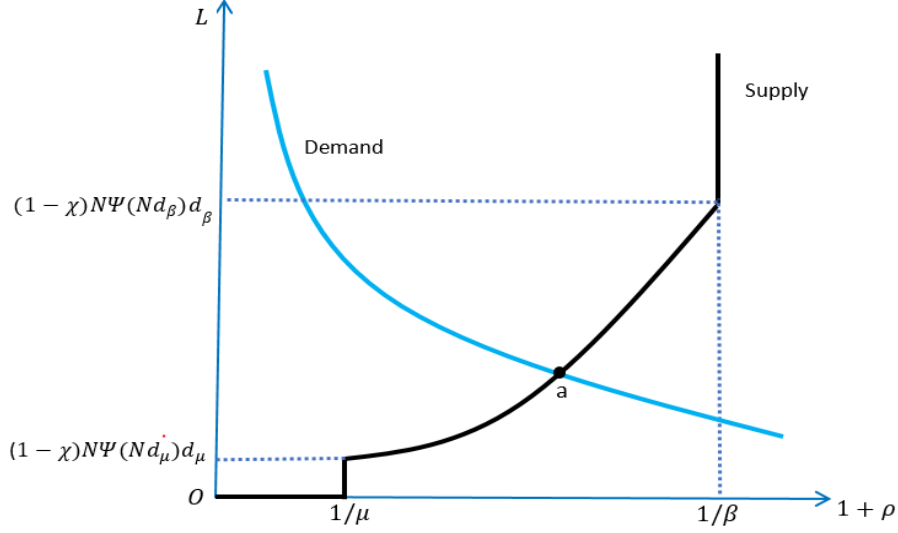
Figure 2: Loan Market Equilibrium

The equilibrium loan supply of an individual bank is

$$
\ell(\rho) = \begin{cases}
0 & if \quad 1+\rho < 1/\mu, \\
[0, (1-\chi)d(\rho)\Psi(Nd(\rho))] & if \quad 1+\rho = 1/\mu, \\
(1-\chi)d(\rho)\Psi(Nd(\rho)) & if \quad 1/\mu < 1+\rho < 1/\beta, \\
[(1-\chi)d_\beta\Psi(Nd_\beta), \infty] & if \quad 1+\rho = 1/\beta,
\end{cases}
$$

Then we obtain the aggregate loan supply of the commerical banks $L^s(\rho) = N\ell(\rho)$. It is represented by the solid black line in Figure 2 in the $(1+\rho)$-L space. If $1+\rho < 1/\mu$, then banks invest only in cash and the aggregate loan supply is zero. If $1+\rho = 1/\mu$, then banks are indifferent between lending and holding cash as long as the reserve requirement does not bind. The aggregate loan supply curve is vertical and can take any value between 0 and $(1-\chi)Nd_\mu\Psi(Nd_\mu)$. On $(1/\mu, 1/\beta)$, the aggregate loan supply curve can in principle be non-monotone, which can be a source for multiple equilibria. However, it is increasing if $D\Psi(D)$ is increasing in D, i.e., the before-interest value of checkable deposits is increasing in their after-interest value. When $1+\rho = 1/\beta$, it is vertical again. Note that if $1+\rho < 1/\beta$, the loans are financed only by checkable deposits, and if $1+\rho = 1/\beta$, the supply beyond the amount financed by checkable deposits is financed by time deposits.

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3.3 Entrepreneurs and the Equilibrium

The entrepreneurs take loan rate ρ as given and choose loan demand to solve

$$\max_{\ell}\{f(\ell) - (1 + \rho)\ell\}.$$

The inverse loan demand for an entrepreneur is $f'(\ell) = 1 + \rho$, which defines the aggregate inverse loan demand function,

$$L^d(\rho) = f'^{-1}(1 + \rho).$$

Obviously $L^d(\cdot)$ is a decreasing function, i.e., the demand for loan decreases with the loan rate. It is always positive and approaches zero (∞) as ρ approaches to ∞ (-1).

The loan demand curve is represented by the solid blue curve in Figure 2. Each of its intersections with the loan supply curve corresponds to a pair of the equilibrium loan rate and loan quantity. After acquiring the equilibrium loan rate ρ^* (we use * to denote equilibrium values), we can plug it into the Cournot solution in Proposition 1 to get equilibrium price and quantity of checkable deposits $\psi^* = \Psi(d(\rho^*))$ and $D^* = Nd(\rho^*)$. The equilibrium is unique if the loan supply curve is non-decreasing, which is guaranteed if $D\Psi(D)$ is increasing. Notice that if entrepreneurs' productivity (or the loan demand curve) is low, then the equilibrium loan rate is equal to the return on cash, i.e, $\rho^* = 1/\mu - 1$. The reserve requirement is loose and banks hold excess reserves.

Proposition 2 *There exists at least one steady-state monetary equilibrium. If, in addition, $D\Psi(D)$ is increasing in D, then the steady-state monetary equilibrium is unique.*

Proof. *See Appendix A.1.* ∎

4 A Baseline CBDC

To illustrate the main mechanism of the paper, we first consider a baseline design of a CBDC: it bears interest, is a perfect substitute for checkable deposits as a means of payment (in type 2 and type 3 meetings), and is not accessible by commercial banks. The supply of the CBDC grows at a gross rate μ_e and pays a nominal interest i_e. The central bank sets μ_e and i_e exogenously. In the next section, we modify the baseline design by allowing the bank to hold the CBDC to satisfy the reserve requirement.[17] In Section 7.2, we consider other

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[17] If banks can hold the CBDC but not as reserves, then the effect of the CBDC remains the same as the baseline design. The intuition is as follows. If the return on the CBDC is less or equal to cash, then banks do not hold the CBDC as assets so the CBDC does not affect the economy. If the return on the CBDC

designs.

With the baseline CBDC, the firm's problem remains the same as before. The household's and bank's problems are different. We first discuss these two problems and then move to the effects on deposits and loans.

4.1   Households

The seller's problem remains unchanged because sellers do not bring any liquidity into the DM. But now a buyer also decides how much CBDC to hold. His/her problem becomes

$$
\begin{aligned}
W^B(z, z_e, d, b) &= z + z_e + d + b + T + \max_x [U(x) - x] \\
&+ \max_{z', z'_e, d', b'} \left\{ -\frac{\phi}{\phi'} z' - \frac{\phi_e}{\phi'_e} \frac{1}{1 + i_e} z'_e - \psi d' - \psi_b b' + \beta V(z', z'_e, d', b') \right\},
\end{aligned}
$$

where z_e is the real balance of the CBDC, and ϕ_e is the price of the CBDC in terms of the CM good. Note that ϕ_e can be different from ϕ in the equilibrium because the CBDC may pay interest or have a different growth rate.

Following steps similar to the case without a CBDC, one can obtain the steady-state household demand for all three payment instruments given the price of deposit ψ and policy rates (i_e, μ_e, μ):

$$
\iota = \frac{\mu}{\beta} - 1 \geq \alpha_1 \lambda(z) + \alpha_3 \lambda(z + z_e + d), \text{ equality iff } z > 0, \tag{8}
$$

$$
\frac{\psi}{\beta} - 1 \geq \alpha_2 \lambda(d + z_e) + \alpha_3 \lambda(z + z_e + d), \text{ equality iff } d > 0, \tag{9}
$$

$$
\frac{\mu_e}{\beta(1 + i_e)} - 1 \geq \alpha_2 \lambda(d + z_e) + \alpha_3 \lambda(z + z_e + d), \text{ equality iff } z_e > 0. \tag{10}
$$

From the last two equations, if $\psi > \frac{\mu_e}{1 + i_e}$, then the demand for checkable deposits is 0. If $\psi < \frac{\mu_e}{1 + i_e}$, then the demand for the CBDC is 0: since the CBDC and checkable deposits are perfect substitutes, the household holds only the instrument that gives a higher rate of return. If $\psi = \frac{\mu_e}{1 + i_e}$, a buyer is indifferent between the CBDC and checkable deposits. He or she cares only about the total electronic payment balances, which include both checkable deposits and the CBDC. Equations (8) to (10) define the inverse demand function for checkable deposits. Denote it as $\hat{\Psi}$ to distinguish from the demand for checkable deposits without a CBDC, Ψ.

---

exceeds cash, then the marginal benefit of investing in the CBDC is negative. To invest in the CBDC, the bank needs to issue deposits and pay at least the CBDC return on its deposits. In addition, the bank must hold cash reserves and invest only a fraction of deposits on the CBDC, which implies that the total cost of investing in the CBDC exceeds the return.
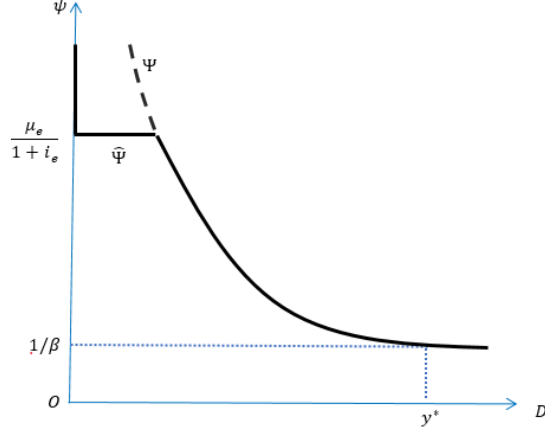
Figure 3: Inverse Demand for Checkable Deposits

Notes. $\Psi(D)$ is the inverse demand for checkable deposits without CBDC, and $\hat{\Psi}(D)$ is the inverse demand for checkable deposits with CBDC.

$$\hat{\Psi}(D) = \begin{cases} [\frac{\mu_e}{1+i_e}, \infty) & D = 0, \\ \frac{\mu_e}{1+i_e} & D \in \left[0, \Psi^{-1}(\frac{\mu_e}{1+i_e})\right], \\ \Psi(D) & D \geq \Psi^{-1}(\frac{\mu_e}{1+i_e}). \end{cases}$$

Figure 3 illustrates how the CBDC changes the demand for checakble deposits. The solid black line is the inverse demand for checkable deposits with the CBDC, while the dashed line is that without the CBDC. They overlap if the price of checkable deposits is below $\mu_e/(1+i_e)$. Once this price exceeds $\mu_e/(1+i_e)$, the demand for checkable deposits drops to 0 after introducing the CBDC.

4.2   Banks

To obtain the bank's problem, simply replace $\Psi(D)$ in (5) by $\hat{\Psi}(D)$. Then we can trace out the aggregate loan supply by solving the Cournot competition for all ρ such that $1 + \rho \in [0, 1/\beta]$. The loan supply function depends on the real gross rate of the CBDC, $(1+i_e)/\mu_e$. Let $r(\rho) = 1/\Psi(Nd(\rho)) - 1$ be the real deposit rate that arises from the Cournot competition without the CBDC. To ease the presentation, assume for now that $1/\mu < (1+i_e)/\mu_e < 1+r_\beta$, where $r_\beta = r(1/\beta - 1)$. We focus on this case because it covers all equilibrium regimes that may occur and is sufficient to illustrate our main mechanism. Appendix A.2 provides a complete analysis of all possible cases.

Let $\bar{\rho}$ satisify $(1 + i_e)/\mu_e = 1 + r(\bar{\rho})$. Because $r(\rho)$ is decreasing under Assumption 1, the

return of the CBDC is lower than the real rate of checkable deposits without the CBDC if $\rho > \bar{\rho}$.[18] Then the equilibrium of the Cournot game stays unchanged after introducing the CBDC because it is strictly dominated by checkable deposits.

Let $\underline{\rho}$ be the loan rate at which banks break even if the deposit rate equals the CBDC rate:

$$(1-\chi)(1+\underline{\rho}) + \chi\frac{1}{\mu} = \frac{1+i_e}{\mu_e}.$$

The left hand side is a bank's revenue from one unit of deposit. It is the sum of the revenue from loans and that from cash reserves, weighed by the reserve requirement ratio. The right hand side is the cost, which is the real gross interest on checkable deposits. If $\rho < \underline{\rho}$, banks cannot compete with the CBDC and they shut down. Then the supply of checkable deposits and loans becomes 0.

If $\underline{\rho} < \rho \leq \bar{\rho}$, the rate of checkable deposits is the same as the CBDC rate and the supply of each bank is $d_e = D_e/N$, where

$$D_e = \Psi^{-1}\left(\frac{\mu_e}{1+i_e}\right). \tag{11}$$

A formal proof can be found in Appendix A.2. Intuitively, if a bank reduces its supply of checkable deposits below d_e, the rate of checkable deposits remains equal to that of the CBDC, because the latter sets a floor for the former. The deviating bank has a strictly lower profit as the marginal profit of checkable deposits is positive, i.e.,

$$(1-\chi)(1+\rho) + \chi\frac{1}{\mu} > \frac{1+i_e}{\mu_e}.$$

Therefore, no banks want to reduce its supply of checkable deposits. On the other hand, no banks want to increase supply because that results in a higher rate on checkable deposits and a lower profit. Notice that by definition, $d(\bar{\rho}) = d_e$ and the deposit supply is continuous at $\rho = \bar{\rho}$.

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[18]From equation (7), under Assumption 1, ψ is weakly decreasing in ρ and strictly decreasing in ρ for $\rho > 1/\mu - 1$. The response of the implied return on checkable deposits $r(\rho)$ is the opposite. Intuitively, under the Cournot competition, a higher return on assets is passed on to the deposit rate, albeit incompletely.

(a) Regime 1: Low i_e (b) Regime 2: Medium low i_e

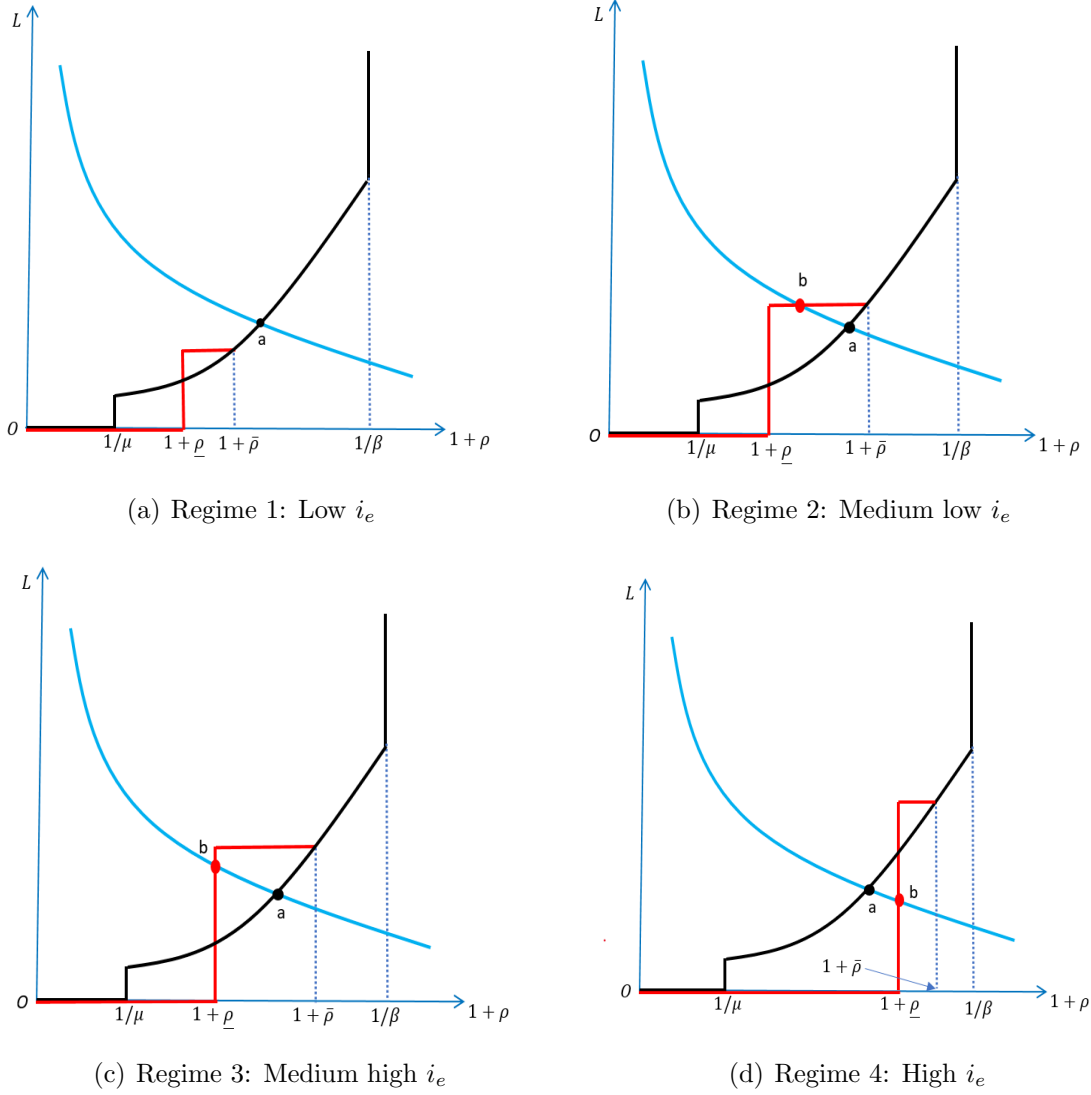(c) Regime 3: Medium high i_e (d) Regime 4: High i_e

Figure 4: Effects of a CBDC

Notes. (1) The blue curve is the loan demand, the black curve is the loan supply without a CBDC, and the red curve is the loan supply with a CBDC. Note that the red curve joins the black curve for $\rho > \bar{\rho}$. (2) The figure illustrates the effect of a CBDC when $1/\mu < (1 + i_e)/\mu_e < 1 + r_\beta$.

If $\rho = \underline{\rho}$, banks are indifferent between any amount of checkable deposits in $[0, d_e]$, as they all lead to 0 profit. Also notice that if $\rho \geq \underline{\rho}$, banks lend up to point at which the reserve requirement is binding.

The above analysis allows us to obtain the aggregate loan supply curve with the baseline CBDC shown by the solid red lines in Figure 4. The black curve is the loan supply curve without a CBDC. Again, we assume that $D\Psi(D)$ is increasing. If $\rho < \underline{\rho}$, the aggregate

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loan supply is 0. If $\rho = \underline{\rho}$, the aggregate supply of checkable deposits can take any value between 0 and D_e. As a result, the aggregage supply of loans can be anything between 0 and $(1-\chi)D_e\Psi(D_e)$. This corresponds to the vertical part of the solid red line. If $\rho \in (\underline{\rho}, \bar{\rho})$, the aggregate supply of loans stays at $(1-\chi)D_e\Psi(D_e)$. This corresponds to the horizontal part of the solid red line. If $\rho > \bar{\rho}$, then the deposit rate offered by banks in the absence of a CBDC is higher than the CBDC rate, and the CBDC does not affect the economy. Therefore, the aggregate loan supply curves with and without a CBDC coincide. As i_e increases, both $\underline{\rho}$ and $\bar{\rho}$ shift to the right. At the same time, the horizontal part of the red curve shifts up as D_e becomes higher.

4.3 Equilibrium

The aggregrage loan demand stays unchanged and is plotted by the solid blue curves in Figure 4. Its intersections with the solid red curve and the solid black curve correspond to equilibria with and without a CBDC.

As the CBDC rate increases, the economy goes through four regimes. Regime 1 is shown in Figure 4(a). It occurs if $\bar{\rho} < \rho^*$. This is equivalent to $i_e < i_{e1}$, where i_{e1} solves $(1+i_e)/\mu_e - 1 = r^*$ and $r^* = r(\rho^*)$ is the equilibrium real rate of checkable deposits without a CBDC. In this regime, the CBDC does not affect the equilibrium.

Once i_e exceeds i_{e1}, the equilibrium switches to regime 2, which is shown in Figure 4(b). Compared with the case without a CBDC, the CBDC raises the deposit rate and the demand for electronic payment balances. If the CBDC were not introduced, banks would have restricted their supply of checkable deposits and offer lower deposit rates. Because the CBDC sets a floor for the rate of checkable deposits, this incentive is no longer active once the floor becomes effective. Moreover, the marginal profit from checkable deposits is positive if their rate equals the CBDC rate. Therefore, banks supply D_e checkable deposits to meet all the demand for the electronic payment balances at the CBDC rate, and the CBDC is not used. A bank invests $1-\chi$ fraction of its checkable deposits on loans, so the aggregate loan quantity is $L_e = (1-\chi)D_e\mu_e/(1+i_e)$. More checkable deposits lead to more loans and a lower loan rate. A higher CBDC rate increases the rate of checkable deposits and D_e. It also increases the loan quantity and decreases the loan rate. Banks have a lower profit margin from checkable deposits because of the higher deposit rate and the lower loan rate. If $i_e = i_{e2}$, which solves $(1-\chi)f'(L_e) + \chi/\mu = (1+i_e)/\mu_e$, or equivalently, $\underline{\rho} = f'(L_e) - 1$, the profit margin reaches 0 and all banks make zero profit.

As i_e increases beyond i_{e2}, the economy enters into regime 3, illustrated in Figure 4(c).

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Then, a higher in i_e increases the rates of checkable deposits and loans. In this regime, the marginal profit from checkable deposits is 0 and banks behave as if they are perfectly competitive. To stay break-even, banks have to increase the loan rate to compensate for a higher deposit rate. This lowers the loan demand and the equilibrium loan quantity. Banks then create fewer checkable deposits to finance loans. However, households increase their electronic payment balances by holding more CBDC. If the CBDC rate is lower than i_{e3}, which solves $(1 - \chi)\rho^* + \chi/\mu = (1 + i_e)/\mu_e$ (or equivalently, $\underline{\rho} = \rho^*$), introducing the CBDC still leads to more loans and deposits.

Finally, as i_e increases beyond i_{e3}, regime 4 occurs. It is the same as regime 3 except the CBDC rate is too high so that the quantities of checkable deposits and loans drop below the level without the CBDC. In other words, the CBDC causes disintermediation if and only if $i_e > i_{e3}$.

The following proposition summarizes these discussions.

Proposition 3 *Suppose that banks cannot hold a CBDC. If $D\Psi(D)$ is increasing, then there exists a unique steady-state monetary equilibrium. As $(1 + i_e)/\mu_e$ increases from $1/\mu$ to $1 + r_\beta$, the effect of the CBDC is as follows:*

1. *if $i_e \leq i_{e1}$, or $\bar{\rho} \leq \rho^*$, then the CBDC does not affect the economy;*

2. *if $i_e \in (i_{e1}, i_{e2})$, or equivalently, $\bar{\rho} > \rho^*$ and $\underline{\rho} < f'(L_e) - 1$, then the CBDC increases lending relative to the case without a CBDC, and a higher i_e induces more lending;*

3. *if $i_e \in (i_{e2}, i_{e3})$, or equivalently, $\bar{\rho} > \rho^*$ and $\underline{\rho} > f'(L_e) - 1$, then the CBDC increases lending relative to the case without a CBDC, and a higher i_e induces less lending;*

4. *if $i_e > i_{e3}$, or equivalently, $\rho^* < \bar{\rho}$, then the CBDC decreases lending relative to the case without a CBDC, and a higher i_e induces less lending.*

Our analysis delivers three important messages. First, introducing a CBDC does not necessarily cause disintermediation and reduce bank loans and deposits. Indeed, the CBDC expands bank intermediation by introducing more competition to the banking sector if its rate falls between i_{e1} and i_{e3}.

Second, one should not judge the effectiveness of the CBDC on its usage, but rather on how much it affects the deposit and the lending rates or quantities. Throughout regime 2, the CBDC is not used, but increases both deposits and loans. In fact, it maximizes lending if $i_e = i_{e3}$, which is in regime 2. Here, the CBDC works as a *potential entrant*. It disciplines

the off-equilibrium outcome: if banks reduce their real deposit rates below the real CBDC rate, then buyers would switch to the CBDC.

Third, there can be a trade-off between payment efficiency and the investment quantity. In regimes 3 and 4, both the CBDC and checkable deposits are used as means of payment. If the CBDC rate is higher, households hold more electronic payment balances. This allows them to consume more in the DM. But at the same time, the loan rate increases and loan quantity falls. For more discussions, see Keister and Sanches (2019).

We end this section with a comparison between a interest floor policy and the CBDC. They share some similarities but the CBDC in general delivers better outcomes. If the CBDC rate i_e is lower than i_{e2}, the economy is in regime 1 and the CBDC is not used. Then the effect of a CBDC is identical to policy that mandates commercial banks to pay a real rate no less than $(1 + i_e)/\mu_e - 1$. If, however, i_e is larger than i_{e2}, households use the CBDC for transactions, because the commercial banks do not create enough checkable deposits to satisfy the transaction needs. With an interest floor policy, the CBDC is not available to households any more. As a result, households have lower electronic payment balances and consume less in the type 2 and 3 meetings. This reduces welfare. In fact, the equilibrium under a CBDC with i_e larger than i_{e2} cannot be achieved by any interest floor policy, because the latter does not provide additional electronic payment balance to meet the demand.

5 CBDC as Reserves

Now we modify the baseline design to allow banks to hold the CBDC as interest-bearing reserves, i.e., it can be used to satisfy the reserve requirement. The CBDC now plays two roles. First, it is a means of payment that competes with checkable deposits. Second, it can lower the cost for the banks to hold reserves if it has a higher return than cash.

The household's and firm's problems remain the same as with the baseline CBDC. The bank's problem changes to

$$
\max_{z_j^e, z_j, \ell_j, d_j} \left\{ (1 + \rho) \, \ell_j + \frac{z_j}{\mu} + \frac{(1 + i_e) \, z_j^e}{\mu_e} - d_j \right\}
$$
$$
\text{s.t.} \quad \ell_j + z_j + z_j^e = \hat{\Psi} \left(d_{-j} + d_j \right) d_j,
$$
$$
z_j^e + z_j \geq \chi \hat{\Psi} \left(d_{-j} + d_j \right) d_j,
$$

where z_j^e is bank j's CBDC balance. As before, we solve the Cournot game among banks for each value of ρ and trace out the aggregate loan supply.

To solve for the equilibrium in this Cournot game, we adopt a two-step approach that parallels the one used for the baseline CBDC. In the first step, we solve the equilibrium of an auxiliary model where only banks can hold the CBDC. This shuts down the role of the CBDC as a competing means of payment. It is similar to the model without CBDC. The only difference is that banks may earn the CBDC rate on the reserves. The second step parallels the analysis in Section 4.2. We compare the real rate of the checkable deposits in the auxiliary model with the real CBDC rate. If the former is higher, the equilibrium of the original Cournot game is the same as the equilibrium of the auxiliary model. Otherwise, we adjust the deposit and loan supply in a similar way as in Section 4.2. The details can be found in Appendix A.3.

The red curves in Figure 5 illustrate the resulting aggregate loan supply curve. Again, we assume that $D\Psi(D)$ is increasing. Same as in Figure 4, the aggregate loan supply curve with the CBDC coincides with the horizontal axis if ρ is low. If ρ is intermediate, the curve is flat. The deposit rate matches the CBDC return, and the quantity of loans is fully determined by the CBDC rate and equals $(1 - \chi)D_e\Psi(D_e)$. If ρ is above a cut-off $\bar{\rho}^R$, the return of the CBDC is lower than that of checkable deposits, and the aggregate loan supply curve is upward sloping.

Compared with the baseline design, besides being an alternative payment method (*payment competition effect*), the CBDC can also reduce the bank's cost to hold reserves (*cost-saving effect*). The two effects together shift the loan supply curve without a CBDC, shown by the solid black curve, to the one with the CBDC. To decompose these two effects, we also plot the aggregate loan demand curve in the auxiliary model, where we shut down the payment competition effect. It is the dashed blue curve on $((1 + i_e)/\mu_e, 1 + \bar{\rho}^R)$ but overlaps with the red curve otherwise. The shift from the solid black curve to the dashed curve captures the cost-saving effect, and the shift from the dashed curve to the red curve captures the payment competition effect.[19]

Similar to the baseline design, the equilibrium can be classified into four regimes. These regimes are separated by three cut-offs in the CBDC rate $i_{e1}^R < i_{e2}^R < i_{e3}^R$. Both the regimes and cut-offs parallel those under the baseline CBDC design.

Figure 5(a) shows regime 1, which occurs if $i_e < i_{e1}^R$. The loan demand curve (the solid blue curve) intersects the loan supply curve with the CBDC in its increasing region. Buyers

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[19]In the increasing part of new (red) loan supply curve ($\rho \in [\bar{\rho}^R, 1/\beta - 1)$), the payment competition effect is muted, and the higher loan supply relative to the old (black) supply curve is due to the cost-saving effect. In contrast, with the baseline CBDC design, the cost-saving effect is absent and the loan supply curves with and without the CBDC coincide with each for $\rho \in (\bar{\rho}, 1/\beta - 1)$.

(a) Regime 1: Low i_e

(b) Regime 2: Medium low i_e

(c) Regime 3: Medium high i_e
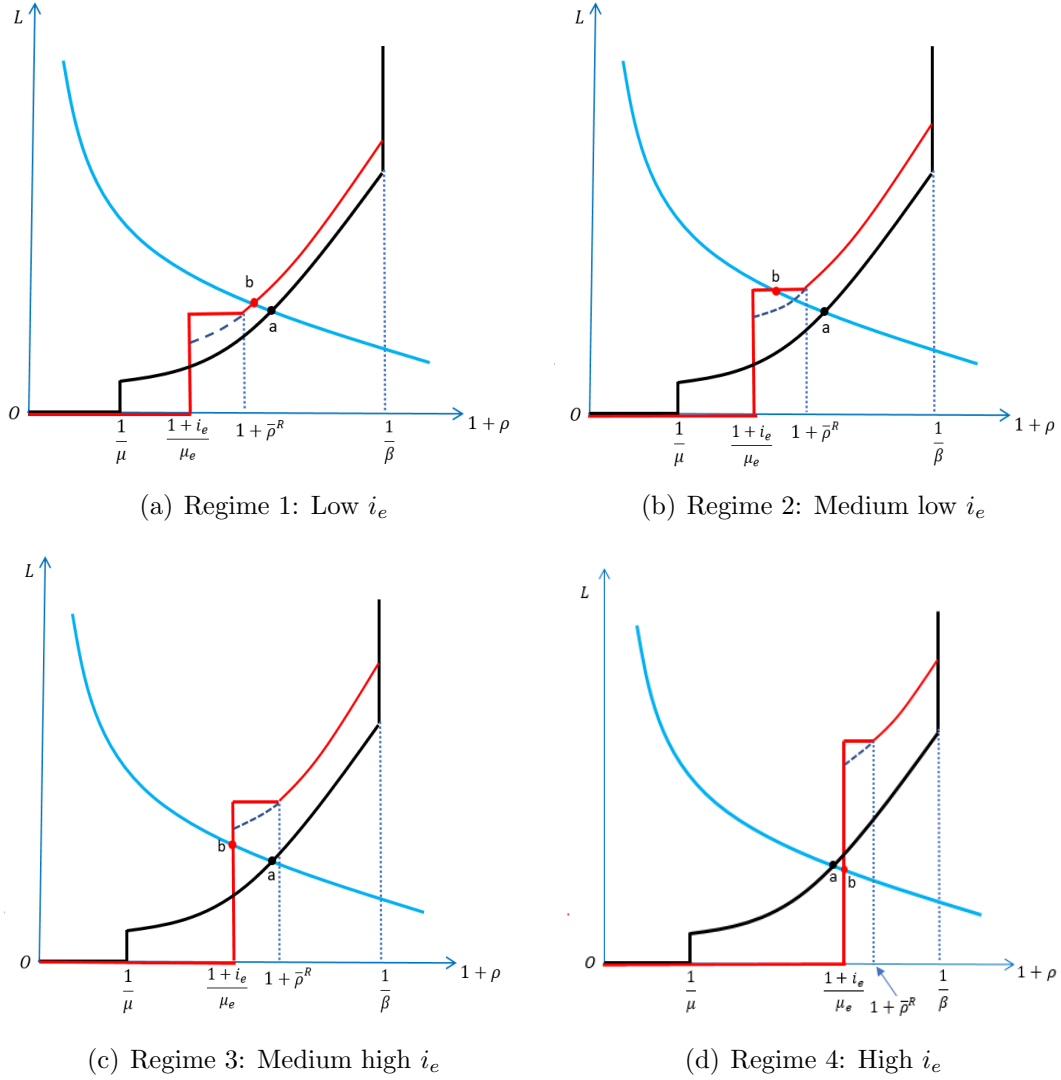
(d) Regime 4: High i_e

Figure 5: CBDC as Reserves

Notes. (1) The blue curve is the loan demand, the black curve is the loan supply without a CBDC, the dashed black line is the loan supply when a CBDC is used as reserves but cannot be used for payments, and the red curve is the final new loan supply with a CBDC that can be used for both reserves and payments. The dashed line coincides with the red curve except for $\rho \in \left((1 + i_e)/\mu_e, \bar{\rho}^R\right]$. All three curves join each other at $1 + \rho = 1/\beta$.

strictly prefer bank deposits to the CBDC and the payment competition effect is not operative. However, the cost-saving effect is still present. If the CBDC offers a higher return than cash, it reduces the cost to hold reserves. As a result, the bank supplies more checkable deposits and loans (point b), and the equilibrium loan rate is lower compared with the equilibrium without the CBDC (point a). In this regime, a higher i_e decreases the cost of holding reserves; hence it increases the deposit rate and the quantities of deposits and loans. It also reduces the loan rate.

As i_e increases beyond i_{e1}^R, the equilibrium switches to regime 2, illustrated by Figure 5(b). Same as in the baseline design, banks issue D_e checkable deposits to absorb all the demand for the electronic payment and the CBDC is not used. As the CBDC rate rise, the equilibrium rate and quantity for checkable deposits rises, the amount of loan supply increases and the loan rate decreases. This reduces the bank's profit margin. Once i_e reaches i_{e2}^R, the bank profit becomes 0.

If i_e exceeds i_{e2}^R, the economy enters into regime 3, shown in Figure 5(c). Same as with the baseline design, banks earn zero profit in this regime. An increase in i_e induces the deposit and loan rates to increase, and the quantity of loans to decrease.[20] Households use both checkable deposits and the CBDC for transactions. As i_e further increases to i_{e3}^R the total loan quantity decreases to the level when there is no CBDC.

As i_e increases beyond i_{e3}^R, the economy enters into regime 4 shown in Figure 5(d). This is the only regime in which the CBDC causes disintermediation relative to the case without a CBDC. As the CBDC rate continues to rise, the response of the economy is the same as in regime 3. In both regime 3 and regime 4, a higher CBDC rate improves payment efficiency at the cost of investment.

To summarize, a CBDC that can serve as reserves can also lead to more bank intermediation. Compared with the baseline CBDC, it has the additional cost-saving effect. This effect can be active and increases lending even if the CBDC has a lower rate of return than checkable deposits. Therefore, this design can increase bank intermediation more than the baseline design. Moreover, the range of i_e in which more bank intermediation arises is wider compared to the baseline CBDC. Also notice that $i_{e1}^R \geq i_{e1}$. Because of the cost-saving effect, banks

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[20]Strictly speaking, the supply of checkable deposits is indeterminate. The bank is indifferent between issuing just enough deposits to support its lending, and meeting the total demand for liquidity with checkable deposits and investing the extra deposits in the CBDC. This indeterminacy disappears when there is a positive proportional fee to manage the deposits (as the cost of issuing deposits will be less than the return on the CBDC investment). So one can consider a refinement where the management cost of checkable deposits converges to 0, which results in the equilibrium where banks just supply enough deposits to finance their lending.

offer a higher rate on checkable deposits than under the baseline design. Then, a higher rate is necessary for the CBDC to be a competitive payment method. Similarly, a higher CBDC rate is needed to drive the bank profit to 0, i.e., $i_{e2}^R \geq i_{e2}$.

6 Quantitative Analysis

Theoretically, the CBDC can increase bank lending if its interest rate lies in a certain range. It remains empirical questions how large this range is and how big the effect of the CBDC can be. These questions are important for policy decisions. We calibrate our model to the U.S. economy between 2014 and 2019, and then conduct a counterfactual analysis to answer these questions. We also study the effects of a non-interest bearing CBDC if the economy trends towards cashless.

6.1 Calibration

We introduce two modifications to the model. First, we assume that banks incur a management cost c per unit of deposits.[21] In our model, this is equivalent to a variable asset management cost. Second, we allow sellers in the DM to have market power. Specifically, in the DM, the terms of trade are determined by Kalai bargaining with bargaining power θ to the buyer. These two modifications do not affect the qualitative analysis but capture two features in the data: sellers have substantial markups and banks have operational costs. Both features can be quantitatively important.

Consider an annual model and the functional forms $U(x) = B \log x$, $u(y) = [(y + \epsilon)^{1-\sigma} - \epsilon^{1-\sigma}]/(1 - \sigma)$, and $f(k) = Ak^\eta$. The parameter ϵ is set to 0.001. It is introduced to guarantee $u(0) = 0$ so that the Kalai bargaining is well-defined for all σ. It has little effect quantitatively. To simplify presentation, define $\Omega = \alpha_1 + \alpha_2 + \alpha_3$ as the trading probability of a buyer.[22] Define $\hat{\alpha}_i = \alpha_i/\Omega, i = 1, 2, 3$, as the probabilities of being in a type i meeting conditional on being matched with a seller. We can then replace α_i with Ω and $\hat{\alpha}_i$. There are 14 parameters to calibrate: $(A, B, N, \Omega, \hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3, \sigma, c, \theta, \beta, \eta, \chi, \mu)$. Eight parameters, $\beta, \eta, \mu, \chi, \hat{\alpha}_i (i = 1, 2, 3)$, and c, are set directly. The rest are calibrated internally. We assume that $(\Omega, B, \sigma, \eta)$ are relatively stable over time. This allows us to use time series data from early years for calibration. While for $\hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3, c, A, N, \mu$, we use only data between 2014 and 2019 because they may vary over the time.

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[21]This adds the term $-c(\psi d + \psi_b b)$ to the profit function in (5).

[22]Since the buyer may not meet a seller in the DM in every period, Ω can be less than 1. One interpretation is that it captures the size of the retail sector in the economy.

We use four data sets in this exercise: (1) Survey of Consumer Payment Choice (SCPC) and Diary of Consumer Payment Choice (DCPC) from the Federal Reserve Bank of Atlanta; (2) call reports data from FFIEC; (3) new M1 series from Lucas and Nicolini (2015); and (4) several macro series from FRED. In what follows, we briefly discuss the calibration of several key parameters. For details, see Appendix E.

We obtain the $\hat{\alpha}$s from SCPC and DCPC from the Federal Reserve Bank of Atlanta. The SCPC contains information on the fraction of online transactions and the DCPC contains information on the perceived fraction of point-of-sale transactions that do not accept cash or debit/credit cards. We use the data from the 2016 wave and results are similar in 2015 and 2017. The SCPC documents that an average household makes 67.8 transactions per month. They include 6.6 automatic bill payments, 5.9 online bill payments and 4.7 online or electronic non-bill payments. We count these as online transactions and they represent 25.37% of all transactions. We assume that all the online transactions accept only deposits. At the point of sale, the DCPC reports that 15% transactions do not accept debit/credit card and 2% transactions do not accept cash. Then, cash-only transactions are those at points of sale that do not accept cards. This implies $\hat{\alpha}_1 = 15\%(1 - 25.37\%) = 11.19\%$. Deposit-only transactions include online transactions and point-of-sale transactions that do not accept cash. Hence, $\hat{\alpha}_2 = 25.37\% + 2\%(1 - 25.37\%) = 26.86\%$. And $\hat{\alpha}_3 = 1 - \hat{\alpha}_1 - \hat{\alpha}_2 = 61.94\%$.

Next, calibrate (Ω, σ, B) using the standard approach of matching the money demand curve. We use the new M1 series from Lucas and Nicolini (2015). This data include checkable deposits and some interest-bearing liquid accounts. To calculate the money demand in the model, we also need the deposit rates. The call reports data, which was also used in Drechsler et. al (2017) and Drechsler et. al (2018), contain balances and interest expenses on transaction accounts. We take the ratio of these two variables to obtain an average interest rate on transaction deposits. For this calibration, we use data from 1987 to 2008. Before 1987, data on interest expenses on transaction accounts are not available. After the financial crises in 2008, there was a sharp rise in the demand for bank notes as a store of value. This motive is not our focus.[23] Notice that in this exercise, we set $\hat{\alpha}$s to their values in 2016. In reality, they may change during 1987-2008 and may be different from their values in 2016. However, our approach remains valid if they change in a way that does not significantly affect the demand for M1, which includes both cash and checkable deposits. The data seem to confirm this assumption, i.e., the money demand curve is stable during 1987-2008.

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[23]We have also done a calibration with data between 1987 and 2019. The increased money demand leads to a larger DM. As a result, the effect of a CBDC is bigger. In this sense, our results can be considered as a lower bound.

Parameters	Notation	Value	Notes
Calibrated externally			
Discount factor	β	0.96	Standard in literature
Curvature of production	η	0.66	Elasticity of commercial loans
Reserve requirement	χ	0.1	U.S. regulation
Cost of handling deposits	c	0.0202	Avg. Operating Cost Per Dollar Asset 2.02%
Gross money growth rate	μ	1.01515	2014-2019 Avg. Annual inflation 1.515%
Frac. of type 1 trades	$\hat{\alpha}_1$	11.19%	SCPC 2016
Frac. of type 2 trades	$\hat{\alpha}_2$	26.86%	SCPC 2016
Frac. of type 3 trades	$\hat{\alpha}_3$	61.94%	SCPC 2016
Calibrated internally			
Prob. of DM trading	Ω	0.24	Money demand 1987-2008
Coeff. on CM consumption	B	2.9967	Money demand 1987-2008
Curv. of DM consumption	σ	1.8083	Money demand 1987-2008
TFP	A	1.4390	Rate on Transaction Accounts 0.3049%
Number of banks	N	26	Spread b/w Transaction Accounts and Loans 3.39%
Buyer's bargaining power	θ	0.9995	Retailer markup 20%

Table 1: Calibration Results

Set η to match the elasticity of commercial loans with respect to the prime rate using the time series from FRED. Given η, choose (A, c, N) to hit several targets in the banking sector between 2014 and 2019. These targets are computed from the call reports data. We choose c to match the average non-interest expenditures excluding expenditures on premises or rent per dollar of assets. Set A to match the average interest rate on transaction accounts. Pick N to match the spread between the loan rate and the rate on transaction accounts.

Table 1 summarizes all the parameter values. Figure 6(a) shows the model-predicted money demand curve against the data between 1987 and 2008. The model fits the data well. Figure 6(b) shows the loan supply and loan demand as functions of $1 + \rho$ under the calibrated parameters. The loan supply curve is monotone and the equilibrium is unique.

6.2 Effects of a CBDC on Banking

Now we introduce a CBDC that is a perfect substitute for checkable deposits. We consider both the baseline design, and the modified design where the CBDC serves as reserves. The supply of CBDC grows at the same rate as cash. We focus on how the CBDC affects the economy with different interest rates. We are particularly interested in lending and output.

Figure 7 shows the results. The first row displays percentage changes of deposits, loans and total output relative to the equilibrium without the CBDC. The second row shows deposit and loan rates and their difference, i.e., the spread. All rates are nominal and in percentages.
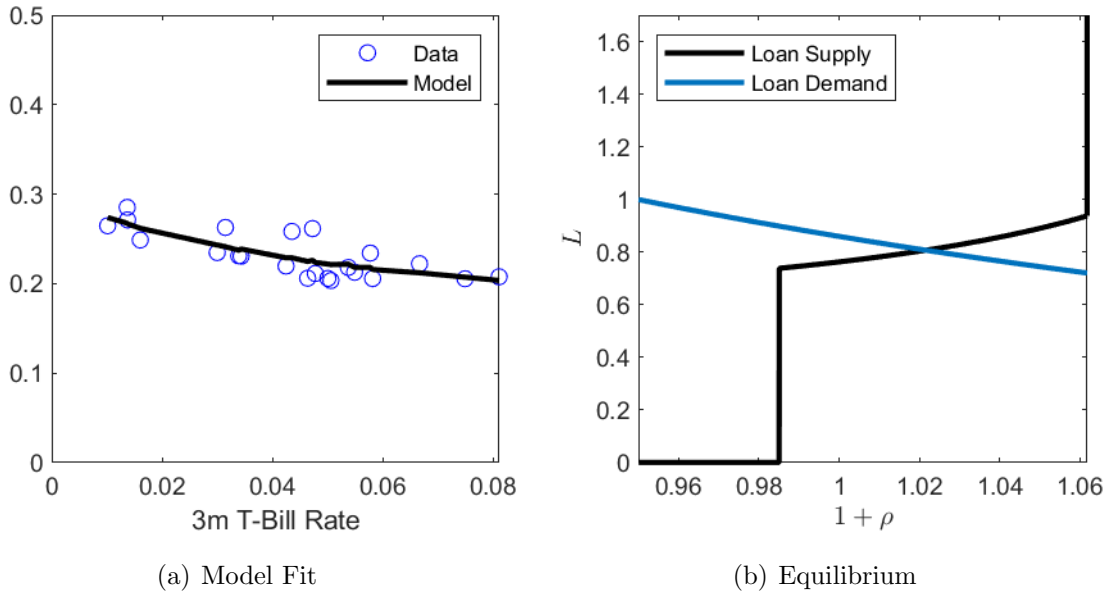
(a) Model Fit   (b) Equilibrium

Figure 6: Money Demand and Equilibrium Under Calibrated Parameters

The blue curve is under the baseline design, and the red curve is under the modified design.

Let us start with the quantities of checkable deposits and loans. First, focus on the economy with the baseline CBDC. If i_e is lower than 0.3049%, which is the deposit rate without the CBDC, the rate of the CBDC is below that of checkable deposits. The economy stays in regime 1. The CBDC is not used and has no effect on the economy. As i_e increases, we see the first kink in the deposit quantity curve, which leads to the first kink in the loan quantity curve. Then the economy enters regime 2. The competition effect is active and leads to more deposits and loans. An increase in i_e significantly increases checkable deposits and loans. If i_e is sufficiently high, then deposits and loans start to decrease and the economy enters regime 3. If checkable deposits and loans drop below their levels without a CBDC, the economy enters regime 4. Relative to the economy without a CBDC, the baseline CBDC increases lending if its rate is between 0.3049% and 1.28%. At the maximum, it increases lending by 1.53%.

If the CBDC also serves as reserves, its effect on deposits and loans is stronger than the baseline CBDC. Because it has the additional cost-saving effect, it has a positive effect on deposits and loans if $i_e > 0$. The red curve is above the blue curve in the first two panels of Figure 7 if $0 < i_e < 0.3049\%$. If i_e becomes larger, the competition effect becomes active. Because banks satisfy all the demand of electronic payment balances at the CBDC rate, the checkable deposit and loan quantities are the same under both CBDC designs. The blue and
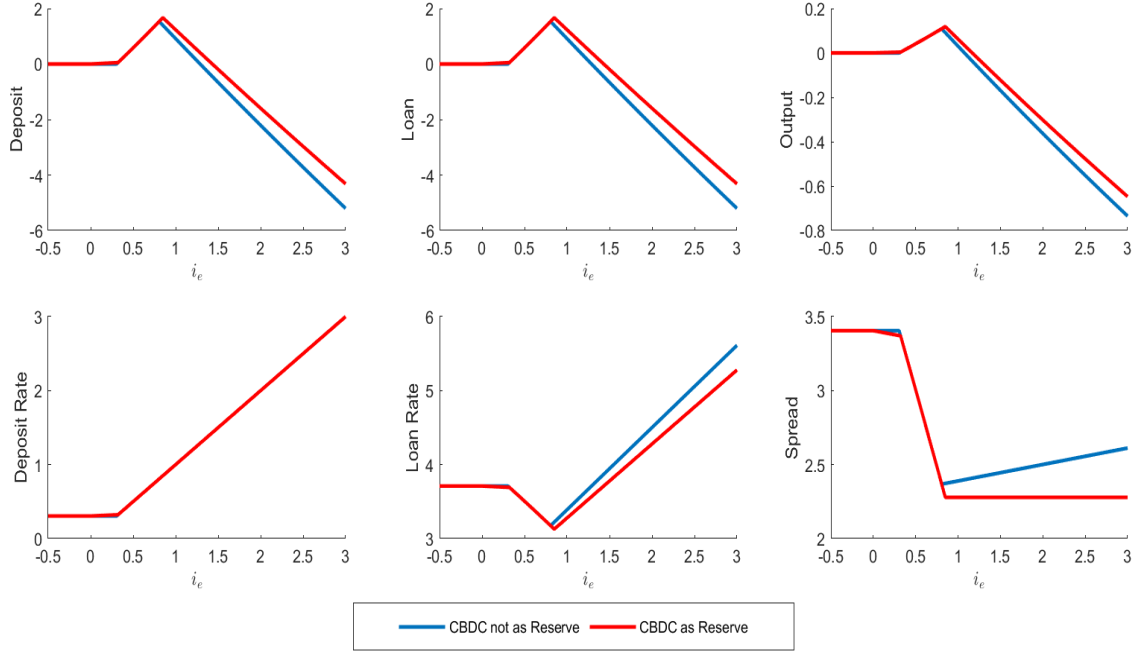
31

Figure 7: Effects of CBDC Rate

the red curves overlap. Because of the cost-saving effect, the red curve declines at a bigger i_e and is always above the blue curve for sufficiently large i_e. Under this design, the CBDC increases bank intermediation if i_e is between 0% and 1.43%. At the maximum, it increases lending by 1.67%

Notice that the competition effect is more important than the cost-saving effect because of two reasons. First, the competition effect applies to all checkable deposits, while the cost-saving effect applies only to the reserves, which is at most 10% of all checkable deposits. Second, banks may not fully pass the cost-saving effect to the economy because of their market power.

Now we turn to the interest rates and the spread. The deposit rate is shown in the first panel in the second row. With the baseline CBDC, the deposit rate is constant if i_e is below 0.3049% and coincides with the 45°-line once the i_e exceeds the deposit rate in the absence of a CBDC. This reflects that the CBDC rate serves as a floor of the deposit rate. If the CBDC serves as reserves, it has the additional cost-saving effect. Therefore, a higher i_e increases the the deposit rate if $0 < i_e < 0.3049\%$. But this effect is very small.

The loan rate reverses the pattern of loans, as shown in the second panel. If i_e is set appropriately, then the loan rate reduces to around 3.17% from about 3.7%. If i_e is too high, then the loan rate can be higher than in the equilibrium without a CBDC, which hurts

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lending.

The spread, defined as the nominal lending rate minus the nominal deposit rate, is shown in the third panel in the second row. The CBDC reduces the spread by competing with checkable deposits. If i_e is sufficiently high, banks act as if the market is perfectly competitive. Then, the lending rate equals the marginal cost of lending, which include the interest paid on deposits, the cost of handling deposits, and the cost of holding reserves.[24] With the baseline CBDC, the spread increases with the CBDC rate. This is caused by the higher cost of holding reserves because the difference between the deposit rate and return on cash reserves increases. This difference disappears if the CBDC serves as reserves, and the spread stays constant for all sufficiently high i_e.[25]

6.3 Effects on Output and Welfare

Now we move to output, which is shown in the third panel in the first row.[26] The pattern is qualitatively similar to loans: as i_e increases, total output first increases and then decreases. Quantitatively, there are two differences. First, introducing a CBDC increases total output for $i_e \in (0.3049\%, 1.07\%)$ if it does not serve as reserves and for $i_e \in (0, 1.16\%)$ if it serves as reserves. Second, the percentage increase in output is much smaller than loans. The highest increase in outptut is 0.108% if the CBDC does not serve as reserves and 0.118% if the CBDC serves as reserves. They are achieved at $i_e = 0.8\%$ and $i_e = 0.85\%$, respectively. The expansionary effect on output is modest relative to lending because of the diminishing return in production.

We next discuss the effect of a CBDC on the welfare of different agents. Figure 8 shows changes in welfare for buyers, sellers, entrepreneurs, and bankers. We measure welfare by the percentage change in consumption that is needed to make an agent indifferent between no CBDC and a CBDC with interest rate i_e. If it is positive, the CBDC increases welfare of the agent. Otherwise, the CBDC reduces welfare. All the y-axes are in percentages.

Buyers and sellers benefit from the CBDC, and their surpluses increase in the range of i_e considered in Figure 8. Without hurting lending, a buyer's surplus can rise up to 0.25% together with a modest increase in a seller's surplus.

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[24]To see this, it helps to rewrite the bank's profit $(1 + \rho)(1 - \chi)\psi d + \chi\psi d\zeta - d - \psi dc$ as $\ell[(1 + \rho - 1/\psi) - \chi/(1 - \chi)(1/\psi - \zeta) - 1/(1 - \chi)c]$, where ζ is the return on reserves.

[25]The difference between the loan and deposit rates is just enough to cover the account cost $1/(1 - \chi)c$. The coefficient $1/(1 - \chi)$ before c reflects the reserve requirement: the bank pays the account fee for all deposits, but can only loan $1 - \chi$ fraction out. In our calibration, $1/(1 - \chi)c = 0.0202\%/0.9 = 0.0244\%$. This implies a nominal spread of $0.0244\% \times 1.01515 = 0.0278\%$.

[26]The output aggregates the output in the DM and the CM. See Appendix E for the formula.

Entrepreneurs benefit from the CBDC because of the lower lending rate and higher loan quantity. Their maximum welfare gain is about 1% with the baseline CBDC and about 1.1% if the CBDC serves as reserves . A CBDC benefits the entrepreneurs if $i_e < 1.28\%$ under the baseline design. This condition changes to $i_e < 1.43\%$ if the CBDC serves as reserves. Banks lose because the CBDC increases competition in the deposit market. If i_e is sufficiently high, they behave as if the market is perfectly competitive. Their profit is reduced to 0, which is a 100% reduction compared with the equilibrium without a CBDC.

6.4 Non-Interest-Bearing CBDC in a Cashless Economy

We have so far focused on an interest-bearing CBDC. However, central banks may be cautious to use the interest on a CBDC as an active tool and consider only a zero-interest CBDC, at least in the initial stage of introducing a CBDC.[27] If the CBDC does not pay interest, can it still have any effect on banking? This section assesses the effect of a zero-interest CBDC as the payment landscape evolves, captured by changes in the αs.

We consider the trend of declining cash usage experienced in many countries. Several central banks consider this trend as an important reason for issuing a CBDC. We capture this trend by converting $\Delta\%$ type 3 meetings to type 2 meetings, i.e., the probability of type 3 meetings changes to $\alpha_3 - \Delta\% \times \alpha_3$ and that of the type 2 meetings changes to $\alpha_2 + \Delta\% \times \alpha_3$. One interpretation is that some brick-and-mortar sellers close their physical stores and sell online. Therefore, more stores accept only deposits. However, such a change can be due to other reasons. For example, some people stop to use or accept cash recently in fear that it may transmit the COVID-19 virus. We evaluate how an economy with and without a CBDC differ as Δ increases.

The blue line in Figure 9 illustrates the results without a CBDC. As Δ increases, checkable deposits become a better payment instrument. Commercial banks gain more market power and reduce the deposit rate. Buyers hold more deposits despite the reduction in rate, because deposits are more useful. As a result, banks issue more deposits and make more loans. The loan rate decreases, but the spread goes up. Higher lending also translates into higher total output.

The red curve shows the economy with a zero-interest CBDC. Because the CBDC has the same rate as cash, the cost-saving effect is not active. Therefore, results are the same

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[27]The Bank of Canada's contingency planning for a CBDC involves a cash-like CBDC that does not pay interest (see https://www.bankofcanada.ca/2020/02/contingency-planning-central-bank-digital-currency). China's DCEP does not pay interest either.
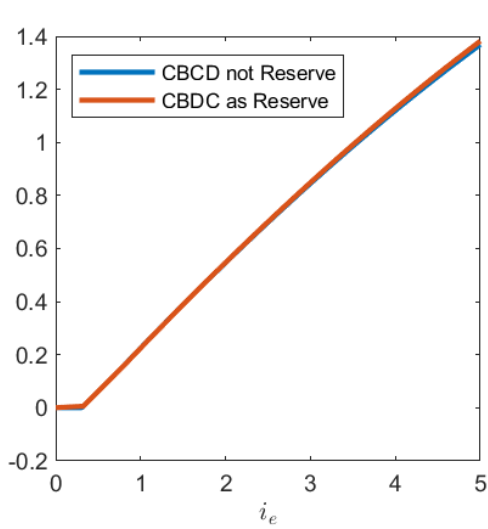
(a) Buyers
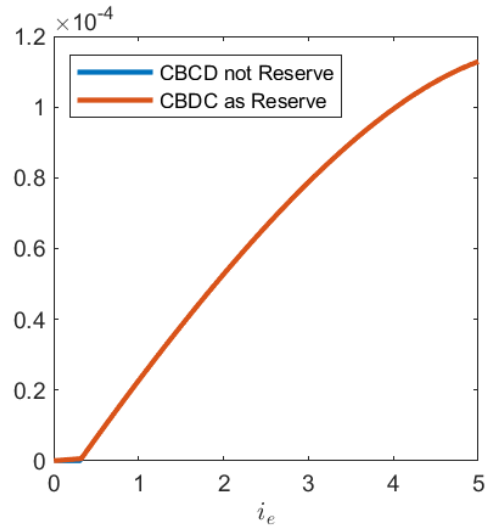
(b) Sellers

(c) Entrepreneurs

(d) Bankers

Figure 8: Welfare Change for Each Type of Agent

Figure 9: Economy Becomes Cashless: $\Delta\%$ Type 3 Meetings Become Type 2 Meetings

regardless whether the CBDC can be used as reserves. If Δ is low, the CBDC rate is lower than the deposit rate. Therefore, it does not affect the equilibrium. As Δ increases, the CBDC prevents the deposit rate from going negative, i.e., 0 becomes a har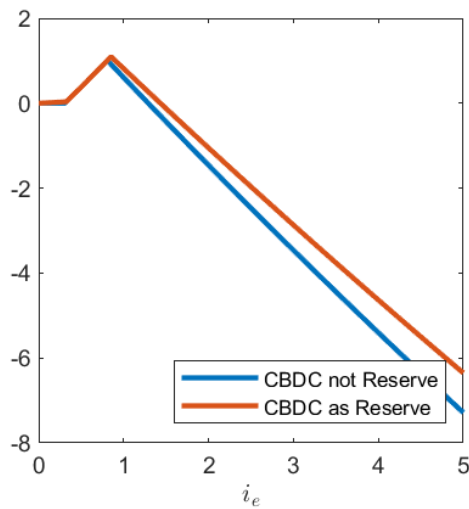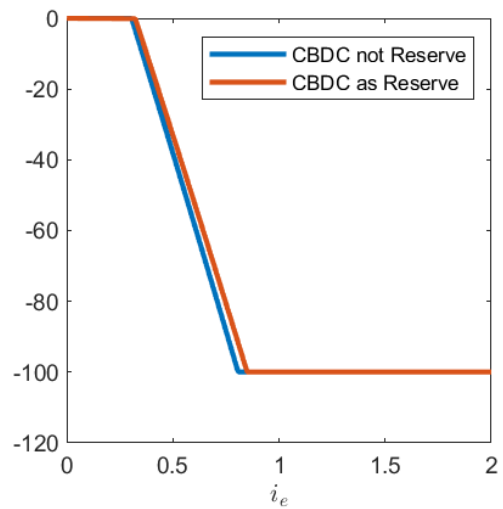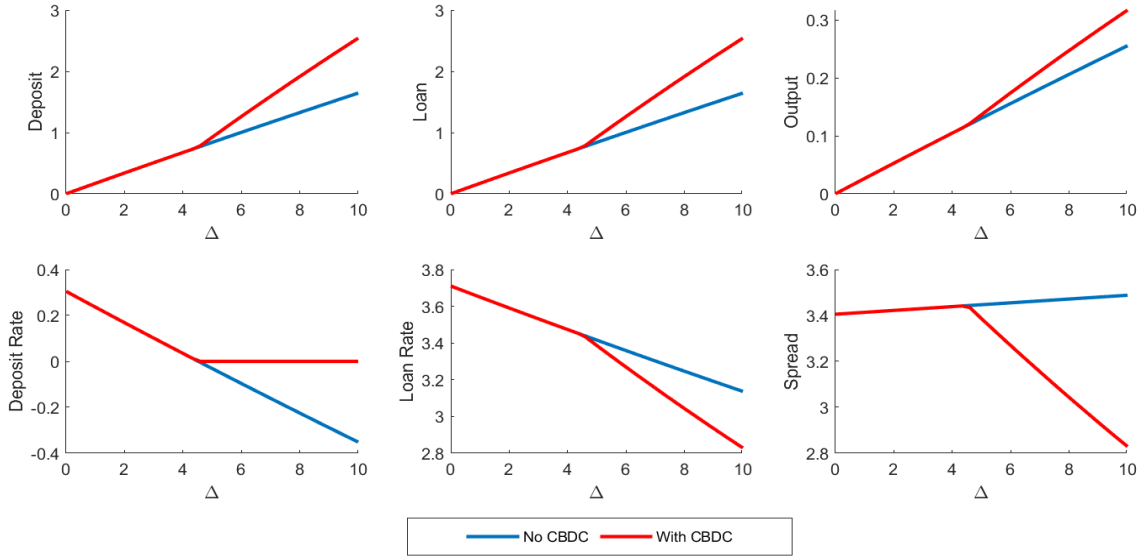d floor of the deposit rate. As a result, banks find it optimal to create more deposits and make more loans. Compared to the case without a CBDC, output is higher and both the loan rate and the spread are lower. If Δ is higher, commerical banks have higher market power. As a result, the CBDC has bigger effects. This exercise shows that if the economy trends toward cashless, a zero-interest CBDC can increase deposits, lending and total output. It is worth noting that a zero-interest CBDC starts to affect the economy if 4.6% of type 3 meetings stop accepting cash. Therefore, the U.S. could reach the situation where a zero-interest CBDC affects the economy with a modest change in the payment landscape.

7 Discussion

7.1 Alternative Modelling Assumptions

The finding that the CBDC can promote bank intermediation is robust with alternative modelling assumptions. We briefly discuss what happens if banks can hold other interest-bearing reserves and if banking sector features endogenous entry. In the Appendix, we also provide detailed analysis of additional extensions including imperfect competition in the loan market, and price competition in the deposit market.

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Interest-Bearing Reserves In the benchmark model, banks hold non-interest-bearing money as reserves. If reserves bear interest, the analysis of the benchmark model stays unchanged except that the term $1/\mu$ in the banker's problem is replaced by $(1 + i_r)/\mu$, where i_r is the interest rate on reserves. Our theoretical analysis stays unchanged and the CBDC still increases deposits and loans with a proper interest rate. The analysis in the benchmark model can be viewed as a special case where $i_r = 0$.

Endogenous Bank Entry We have so far assumed that the number of banks is fixed. Now suppose that many potential banks decide whether to pay a fixed cost to enter the market. If banks have market power only in the deposit market, the results of the paper are unaffected if i_e is not too big. The number of banks may decrease because the CBDC reduces bank profit. However, active banks still satisfy all the demand for electronic payment balances and lend up to the reserve requirement constraint. Therefore, the CBDC can still increase deposits and loans.

7.2   Alternative Designs of a CBDC

We have so far considered a CBDC that serves as a perfect substitute for checkable deposits as a payment instrument. The central bank determines its growth rate μ_e, its interest rate i_e, and whether it can be used as reserves. In this section, we consider other dimensions of the CBDC design, and discuss how they affect our analysis.

Fixed Supply of a CBDC We have so far assumed that the central bank fixes the rate of the CBDC, and its quantity is endogenously determined in equilibrium. Now suppose that the central bank fixes the quantity of the CBDC instead and announces it publicly at the beginning of each period. The rates on the CBDC and deposits are then endogenously determined. Banks then take the quantity of the CBDC as given and solve a similar maximization problem. It is easy to see that in this case, the central bank acts like a new bank entering the deposit market to compete with incumbent bankers. Each commercial bank then creates fewer deposits and makes fewer loans. The total electronic payment balances (checkable deposits plus the CBDC) increase. Competition also forces commerical banks to raise the deposit rate. But different from the previous findings, the loan rate is now always higher, and the CBDC always has a positive market share. The CBDC always causes disintermediation in the banking sector.

CBDC as a Cash Substitute Next consider a CBDC that is the same as the baseline design except that it is a perfect substitute for cash, i.e., it can only be used in type 1 and

type 3 meetings. If it bears a positive nominal interest rate, it dominates cash as a payment instrument. Therefore, it drives cash out of the payment market. Banks still hold cash as reserves. The CBDC is a better competitor to checkable deposits than cash if it offers positive interest. Facing the increasing competition, banks may respond by increasing or decreasing their checkable deposits and loans, depending on the elasticity of the demand for checkable deposits. In general, the level of bank intermediation may increase or decrease.

8 Conclusion

This paper develops a model with imperfect competition in the deposit market to analyze whether introducing a CBDC would cause disintermediation in banks. We show that, contrary to the common wisdom, the CBDC can promote bank intermediation. Intuitively, if banks have market power, they restrict the deposit supply to lower the deposit rate. An interest-bearing CBDC introduces more competition, which leads to the creation of more deposits and lending and a lower loan rate. However, more intermediation happens only if the interest rate on the CBDC lies in some intermediate range. If the CBDC rate is too low, then the CBDC does not affect the equilibrium. If the CBDC rate is too high, disintermediation occurs.

A quantitative analysis using the US data finds that a CBDC that is a perfect substitute for bank deposits as a payment instrument expands bank intermediation if the CBDC rate lies between 0.3049% and 1.28%. At the maximum, it can increase loans and deposits by 1.53% and the total output by 0.108%.

Our model is useful for analyzing effects of CBDCs with various design choices: interest-bearing or not, cash-like or deposit-like, serving as reserves or not, with a fixed quantity or rate, etc. It can also be used to study the role of a CBDC in an increasingly cashless world, and the interaction between CBDC-related policies and existing monetary policy instruments, such as interest on reserves.

Our model abstracts from some important issues related to a CBDC. For example, an interest-bearing CBDC would increase banks' funding costs. On the asset side, banks may invest in more risky projects to make up for their lower profit margin. This can increase the total risk in the financial system. On the liability side, banks may switch to other funding sources, such as wholesale funding. These sources are generally less stable than deposits. Therefore, the CBDC may increase the likelihood of runs in the wholesale market. We leave these issues for future research.

Appendices

A Omitted Proofs and Calculations

A.1 Equilibrium without a CBDC

Proof of Proposition 1. Without loss of generality, rewrite bank j's problem regarding checkable deposits as

$$\max_{d_j} \xi\Psi\left(D_{-j} + d_j\right)d_j - d_j,$$

where $\xi = \max\{1/\mu, (1+\rho)(1-\chi) + \chi/\mu\}$ is the gross return on checkable deposits. By Assumption 1(a), this problem has a unique solution. It satisfies $\Psi'\left(D_{-j} + d_j\right)d_j + \Psi\left(D_{-j} + d_j\right) = 1/\xi$. Then the symmetric pure strategy Nash equilibrium d must satisfy

$$\Psi'\left(Nd\right)d + \Psi\left(Nd\right) = 1/\xi. \tag{12}$$

Because $\Psi' \leq 0$ and $\Psi\left(y^*\right) = \beta < 1/\xi$, $\Psi'\left(Nd\right)d + \Psi\left(Nd\right) < 1/\xi$ if $d = y^*/N$. By Assumption 1(b), equation (12) has a unique solution denoted by $d(\rho)$. Morevoer, because $1/\xi$ is decreasing in ρ, $d(\rho)$ increases with ρ. ∎

We now derive other equilibrium quantities in the Cournot game as functions of ρ. A bank's real balances in cash is

$$z(\rho) = \begin{cases} d(\rho)\Psi(Nd(\rho)) & if \quad 1+\rho < 1/\mu, \\ [\chi d(\rho)\Psi(Nd(\rho)), d(\rho)\Psi(Nd(\rho)))] & if \quad 1+\rho = 1/\mu, \\ \chi d(\rho)\Psi(Nd(\rho)) & if \quad 1+\rho > 1/\mu. \end{cases}$$

The real price of checkable deposits is $\psi(\rho) = \Psi(Nd(\rho))$. The supply of time deposits $b(\rho)$ is

$$b(\rho) = \begin{cases} 0 & if \quad 1+\rho < 1/\beta, \\ [0,\infty] & if \quad 1+\rho = 1/\beta. \end{cases}$$

Proof of Proposition 2. The loan supply $L^s(\rho)$ is continuous. It is increasing on $[1/\mu - 1, 1/\beta - 1)$ if $\Psi(D)D$ is increasing, because $d(\rho)$ is increasing. Moreover, it is 0 if $\rho < 1/\mu - 1$, is ∞ if $\rho > 1/\beta - 1$, and ranges from $(1-\chi)d_\beta\Psi(Nd_\beta)$ to ∞ if $\rho = 1/\beta - 1$. On the other hand, $L^d\left(\rho\right)$ is continuous and decreasing for any $\rho > -1$, with $L^d(-1) = \infty$ and $L^d(\infty) = 0$. As a result, $L^d(\rho) > L^s(\rho)$ for sufficiently small ρ and $L^d(\rho) < L^s(\rho)$ for sufficiently big ρ. By the intermediate value theorem, there exists a unique equilibrium and the loan rate falls

in $(0, 1/\beta - 1]$. ∎

A.2 The Baseline CBDC

This section solves the Cournot game with the baseline CBDC and obtains a bank's equilibrium strategies as functions of ρ. First focus on the quantity of checkable deposits $\hat{d}(\rho)$. Let $r_\beta = r(1/\beta - 1)$. In the main text, we have derived that if $1/\mu < (1 + i_e)/\mu_e < 1 + r_\beta$,

$$\hat{d}(\rho) = \begin{cases} 0 & if \quad \rho < \underline{\rho}, \\ [0, d_e] & if \quad \rho = \underline{\rho}, \\ d_e > d(\rho) & if \quad \rho \in (\underline{\rho}, \bar{\rho}], \\ d(\rho) & if \quad \rho \in (\bar{\rho}, 1/\beta - 1]. \end{cases} \tag{13}$$

By definition, $\underline{\rho}$ is the lowest lending rate at which a bank can break even if it lends up to the reserve requirement. And $\bar{\rho}$ is the lowest lending rate at which banks offer a real deposit rate equal to the $(1 + i_e)/\mu_e - 1$ without CBDC. The condition $1/\mu < (1 + i_e)/\mu_e < 1 + r_\beta$ guarantees $1/\mu - 1 < \underline{\rho} < \bar{\rho} < 1/\beta - 1$. Therefore, all four branches exist. In general, both cut-offs can be lower than $1/\mu - 1$ or over $1/\beta - 1$ and cause some branches to disappear.

Case 1. If $(1 + i_e)/\mu_e < 1 + r_\mu$, where $r_\mu = r(1/\mu - 1)$, then $\bar{\rho} < 1/\mu - 1$. Equation (13) implies that $\hat{d}(\rho) = d(\rho)$ if $\rho \geq 1/\mu - 1$. Moreover, $\hat{d}(\rho)$ and $d(\rho)$ are both constant on $[0, 1/\mu - 1]$ because banks can hold only cash reserves and earn a return of $1/\mu - 1$. Therefore, $\hat{d}(\rho) = d(\rho)$ on $[0, 1/\beta - 1]$. We only have the last branch in (13). Note that without a CBDC, the lowest rate for checkable deposits is given by r_μ.[28] If the CBDC rate is below this value, then it will not affect the economy irrespective of the value of ρ.

Case 2. If $1 + r_\mu \leq (1 + i_e)/\mu_e < 1/\mu$, then $1/\beta - 1 > \bar{\rho} \geq 1/\mu - 1$ and $\underline{\rho} < 1/\mu - 1$. Again, because $\hat{d}(\rho)$ is constant if $\rho \leq 1/\mu - 1$, $\hat{d}(\rho) = d_e > d(\rho)$ if $\rho \leq \bar{\rho}$. Then the last two branches in (13) appear.

Case 3. If $(1 + i_e)/\mu_e = 1/\mu$, then $1/\beta - 1 > \bar{\rho} > 1/\mu - 1$ and $\underline{\rho} = 1/\mu - 1$. Because $\hat{d}(\rho)$ is constant if $\rho \leq 1/\mu - 1$, $\hat{d}(\rho) = d(1/\mu - 1) = [0, d_e]$ if $\rho \leq \underline{\rho} = 1/\mu - 1$. Then the last three branches in (13) appear.

Case 4. If $1/\mu < (1 + i_e)/\mu_e < 1 + r_\beta$, then we are back to the case described in the main paper and $\hat{d}$ is given by (13).

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[28] When $1 + \rho < 1/\mu$, banks invest only in cash, so the return of the bank's asset is bounded below by $1/\mu$, and the deposit rate without a CBDC remains the same as when $1 + \rho = 1/\mu$.

Case 5. If $1 + r_\beta \leq (1 + i_e)/\mu_e < (1 - \chi)/\beta + \chi/\mu$, then $\bar{\rho} \geq 1/\beta > \underline{\rho}$. By equation (13), $\hat{d}(\rho) = 0$ if $\rho < \underline{\rho}$ and $\hat{d}(\rho) = d_e > d(\rho)$ if $\rho \in [\underline{\rho}, 1/\beta - 1]$. The first three branches in (13) appear.

Case 6. If $(1 + i_e)/\mu_e = (1 - \chi)/\beta + \chi/\mu$, then $\bar{\rho}$ is not defined and $\underline{\rho} = 1/\beta$. Then we only have the first two branches of (13).

Case 7. If $(1 + i_e)/\mu_e > (1 - \chi)/\beta + \chi/\mu$, then $\bar{\rho}$ is not defined and $\underline{\rho} > 1/\beta$. In this case, the required rate for checkable deposits is higher than the highest possible return on the bank's assets. As a result, the bank does not offer any checkable deposits for all $\rho \in (0, 1/\beta - 1]$, i.e., only the first branch in (13) appears. For $\rho = 1/\beta - 1$, the bank could still offer time deposits.

Proposition 4 *Suppose Assumption 1 holds and $1 + r_\beta > 1/\mu$. With the baseline CBDC:*

1. *If $(1 + i_e)/\mu_e < 1 + r_\mu$, then the CBDC does not affect the economy for any $\rho \in (0, 1/\beta]$, and $\hat{d}(\rho) = d(\rho)$.*

2. *If $1 + r_\mu < (1 + i_e)/\mu_e < 1/\mu$, then the CBDC affects the economy iff $\rho < \bar{\rho}$, and*

$$\hat{d}(\rho) = \begin{cases} d_e > d(\rho) & if \quad \rho < \bar{\rho}, \\ d(\rho) & if \quad \rho \in (\bar{\rho}, 1/\beta - 1]. \end{cases}$$

3. *If $1/\mu = (1 + i_e)/\mu_e$, then the CBDC affects the economy iff $\rho < \bar{\rho}$, and*

$$\hat{d}(\rho) = \begin{cases} [0, d_e] & if \quad \rho \leq 1/\mu - 1 = \underline{\rho}, \\ d_e > d(\rho) & if \quad \rho \in (\underline{\rho}, \bar{\rho}], \\ d(\rho) & if \quad \rho \in (\bar{\rho}, 1/\beta - 1]. \end{cases}$$

4. *If $1/\mu < (1 + i_e)/\mu_e < 1 + r_\beta$, then the CBDC affects the economy iff $\rho < \bar{\rho}$, and $\hat{d}$ is given by (13).*

5. *If $1 + r_\beta \leq (1 + i_e)/\mu_e < (1 - \chi)/\beta + \chi/\mu$, then the CBDC rate affects the economy for all $\rho \in (0, 1/\beta - 1]$, and*

$$\hat{d}(\rho) = \begin{cases} 0 & if \quad \rho < \underline{\rho}, \\ [0, d_e] & if \quad \rho = \underline{\rho}, \\ d_e > d(\rho) & if \quad \rho \in (\underline{\rho}, 1/\beta - 1]. \end{cases}$$

6. If $(1 + i_e)\mu_e = (1 - \chi)/\beta + \chi/\mu$, then the CBDC rate affects the economy for all $\rho \in (0, 1/\beta - 1]$, and

$$\hat{d}(\rho) = \begin{cases} 0 & if \quad \rho < 1/\beta - 1 = \underline{\rho}, \\ [0, d_e] & if \quad \rho = 1/\beta - 1. \end{cases}$$

Banks are also willing to supply any positive among of time deposits if $\rho = 1/\beta - 1$.

7. If $(1 + i_e)/\mu_e > (1 - \chi)/\beta + \chi/\mu$, then the CBDC rate affects the economy for all $\rho \in (0, 1/\beta - 1]$, and $\hat{d}(\rho) = 0$ for all $\rho \in (0, 1/\beta - 1]$.

Proof. We only prove that $\hat{d}(\rho) = d_e$ if $\rho \in \left(\underline{\rho}, \bar{\rho}\right]$ and $\underline{\rho} > 1/\mu - 1$. The other parts are obvious. First, if total supply of checkable deposits D is lower than $Nd_e = D_e$, then increasing d_j does not change the price of the deposit, which is fixed at $\mu_e/(1 + i_e)$. By the definition of $\underline{\rho}$, the first-order derivative with respect to d_j is

$$[(1 + \rho)(1 - \chi) + \chi/\mu]\frac{\mu_e}{1 + i_e} - 1 > 0,$$

if $\rho > \underline{\rho}$. Therefore, bank j can always increase its profit by increasing d_j.

By the definition of $\bar{\rho}$,

$$[(1 + \bar{\rho})(1 - \chi) + \chi/\mu]\left[\Psi(D_e) + \Psi'(D_e)D_e/N\right] - 1 = 0$$

If $D > D_e$, then by Assumption 1, the marginal profit of a bank

$$[(1 + \rho)(1 - \chi) + \chi/\mu]\left[\Psi(D) + \Psi'(D)D/N\right] - 1 < 0$$

for all $\rho < \bar{\rho}$. It is profitable for a bank to reduce its supply of deposit if $D > Nd_e$. Combining both arguments, banks supply Nd_e checkable deposits in total. Then $\hat{d}(\rho) = d_e$ by symmetry. ∎

To derive the loan supply strategy, just notice that if $\rho < 1/\mu$, a bank makes no loans; if $\rho > 1/\mu - 1$, a bank lends up to the reserve requirement; and if $\rho = 1/\mu - 1$, a bank is indifferent among any loan quantity that satisfies the reserve requirement. Then we can obtain the following. If $(1 + i_e)/\mu_e < 1 + r_\mu$, the CBDC does not affect the loan supply and

$\hat{\ell}(\rho) = \ell(\rho)$. If $1 + r_\mu \leq (1 + i_e)/\mu_e \leq 1/\mu$, then

$$
\hat{\ell}(\rho) = \begin{cases}
0 & if \quad \rho < 1/\mu - 1, \\
\left[0, (1 - \chi)\frac{\mu_e}{1+i_e}d_e\right] & if \quad \rho = 1/\mu - 1, \\
(1 - \chi)\frac{\mu_e}{1+i_e}d_e > \ell(\rho) & if \quad \rho \in (1/\mu - 1, \bar{\rho}], \\
(1 - \chi)\psi(\rho)d(\rho) = \ell(\rho) & if \quad \rho \in (\bar{\rho}, 1/\beta - 1), \\
[(1 - \chi)\psi(\rho)d(\rho), \infty) = \ell(\rho) & if \quad \rho = 1/\beta - 1.
\end{cases}
$$

If $1/\mu < (1 + i_e)/\mu_e < 1 + r_\beta$, then

$$
\hat{\ell}(\rho) = \begin{cases}
0 & if \quad \rho < \underline{\rho}, \\
\left[0, (1 - \chi)\frac{\mu_e}{1+i_e}d_e\right] & if \quad \rho = \underline{\rho}, \\
(1 - \chi)\frac{\mu_e}{1+i_e}d_e > \ell(\rho) & if \quad \rho \in (\underline{\rho}, \bar{\rho}], \\
(1 - \chi)\psi(\rho)d(\rho) = \ell(\rho) & if \quad \rho \in (\bar{\rho}, 1/\beta - 1), \\
[(1 - \chi)\psi(\rho)d(\rho), \infty) = \ell(\rho) & if \quad \rho = 1/\beta - 1.
\end{cases}
$$

If $1 + r_\beta \leq (1 + i_e)/\mu_e < (1 - \chi)/\beta + \chi/\mu$, then

$$
\hat{\ell}(\rho) = \begin{cases}
0 & if \quad \rho < \underline{\rho}, \\
\left[0, (1 - \chi)\frac{\mu_e}{1+i_e}d_e\right] & if \quad \rho = \underline{\rho}, \\
(1 - \chi)\frac{\mu_e}{1+i_e}d_e > \ell(\rho) & if \quad \rho \in (\underline{\rho}, 1/\beta - 1), \\
\left[(1 - \chi)\frac{\mu_e}{1+i_e}d_e, \infty\right) & if \quad \rho = 1/\beta - 1.
\end{cases}
$$

If $(1 + i_e)/\mu_e \geq (1 - \chi)/\beta + \chi/\mu$, then

$$
\hat{\ell}(\rho) = \begin{cases}
0 & for all \ \rho \in (0, 1/\beta - 1), \\
[0, \infty) & if \ \rho = 1/\beta - 1.
\end{cases}
$$

A.3   CBDC as Reserves

We focus only on the case where $1/\mu < (1 + i_e)/\mu_e < 1/\beta$. In the first step, we analyze the equilibrium from the Cournot competition in the auxiliary model. Denote a bank's supply of checkable deposits and loans as $d^R(\rho)$ and $\ell^R(\rho)$, respectively, where the superscript "R" stands for reserves. The real price and rate of checkable deposits are $\psi^R(\rho)$ and $r^R(\rho)$, respectively. In the second step, we compare $(1 + i_e)/\mu_e$ with $r^R(\rho)$ and obtain the bank's supply of checkable deposits, $\hat{d}^R(\rho)$, and the supply of loans, $\hat{\ell}^R(\rho)$ in the original model.

The first step is the same as the analysis in Section 3 except that we replace μ in the bank's problem replaced by $\mu_e/(1+i_e)$. Therefore, $d^R(\rho)$ satisfies

$$\Psi'(Nd^R(\rho))d^R(\rho) + \Psi(Nd^R(\rho)) = \min\left\{\frac{\mu_e}{1+i_e}, \frac{1}{(1+\rho)(1-\chi) + \chi(1+i_e)/\mu_e}\right\}.$$

And the loan supply is

$$\ell^R(\rho) = \begin{cases} 0 & if \quad 1+\rho < (1+i_e)/\mu_e, \\ [0, (1-\chi)d^R(\rho)\Psi(Nd^R(\rho))] & if \quad 1+\rho = (1+i_e)/\mu_e, \\ (1-\chi)d^R(\rho)\Psi(Nd^R(\rho)) & if \quad (1+i_e)/\mu_e < 1+\rho < 1/\beta, \\ [(1-\chi)d^R(\rho)\Psi(Nd^R(\rho)), \infty] & if \quad 1+\rho = 1/\beta. \end{cases}$$

The real price of checkable deposits is $\psi^R(\rho) = \Psi(Nd^R(\rho))$, and the real rate is $r^R(\rho) = 1/\Psi(Nd^R(\rho)) - 1$.

Step 2 is also similar to the case with the baseline CBDC design. Define r_β^R to be $r^R(\rho)$ evaluated at $\rho = 1/\beta - 1$. If $(1+i_e)/\mu_e < 1 + r_\beta^R$, then

$$\hat{d}^R(\rho) = \begin{cases} [0, d_e] & if \quad \rho \leq (1+i_e)/\mu_e - 1, \\ d_e > d^R(\rho) & if \quad \rho \in \left((1+i_e)/\mu_e - 1, \bar{\rho}^R\right], \\ d^R(\rho) & if \quad \rho \in \left(\bar{\rho}^R, 1/\beta - 1\right]. \end{cases} \tag{14}$$

The loan supply is

$$\hat{\ell}^R(\rho) = \begin{cases} 0 & if \quad \rho < (1+i_e)/\mu_e - 1, \\ \left[0, (1-\chi)\frac{\mu_e}{1+i_e}d_e\right], & if \quad \rho = (1+i_e)/\mu_e - 1 \\ (1-\chi)\frac{\mu_e}{1+i_e}d_e > \ell^R(\rho) & if \quad \rho \in \left((1+i_e)/\mu_e - 1, \bar{\rho}^R\right], \\ (1-\chi)\psi^R(\rho)d^R(\rho) = \ell^R(\rho) & if \quad \rho \in \left(\bar{\rho}^R, 1/\beta - 1\right), \\ [(1-\chi)\psi^R(\rho)d^R(\rho), \infty) = \ell^R(\rho) & if \quad \rho = 1/\beta - 1, \end{cases}$$

where $\bar{\rho}^R$ solves $1 + r^R(\bar{\rho}^R) = (1+i_e)/\mu_e$. The intuition of these equations is similar to the case with the baseline CBDC. If $\rho > \bar{\rho}^R$, the competition effect is not active and the quantity of checkable deposits and loans are the same as in the auxiliary model. If $\rho < (1+i_e)/\mu_e - 1$, the supply for checkable deposits lies between zero and d_e. This is different from the baseline CBDC becasue of the additional cost-saving effect. Banks can break even by issuing checkable deposits and hold the CBDC even if ρ is low. One can purify the equilibrium by adding a marginal cost for handling deposits and let the marginal cost converge to 0. In the limit,

the deposit supply is 0 if $\rho < (1+i_e)/\mu_e - 1$. This purification does not affect the loan supply function. Banks supply 0 loans because the return is lower than that of the CBDC. If $\rho \in \left((1+i_e)/\mu_e - 1, \bar{\rho}^R\right]$, banks fully satisfy the household demand for electronic payment balances and lend up to the reserve requirement. If $\rho = 1/\beta$, the bank may also issue time deposits and the loans quantity can be any value larger than that financed by the checkable deposits.

If $1 + r_\beta^R \leq (1+i_e)/\mu_e < 1/\beta$, then

$$\hat{d}^R(\rho) = \begin{cases} [0, d_e] & if \quad \rho \leq (1+i_e)/\mu_e - 1, \\ d_e > d^R(\rho) & if \quad \rho \in ((1+i_e)/\mu_e - 1, 1/\beta - 1], \end{cases}$$

$$\hat{\ell}^R(\rho) = \begin{cases} 0 & if \quad \rho < (1+i_e)/\mu_e - 1, \\ \left[0, (1-\chi)\frac{\mu_e}{1+i_e}d_e\right], & if \quad \rho = (1+i_e)/\mu_e - 1, \\ (1-\chi)\frac{\mu_e}{1+i_e}d_e > \ell^R(\rho) & if \quad \rho \in ((1+i_e)/\mu_e - 1, 1/\beta - 1), \\ \left[(1-\chi)\frac{\mu_e}{1+i_e}d_e, \infty\right) & if \quad \rho = 1/\beta - 1. \end{cases}$$

We have fewer branches compared to the case with $(1+i_e)/\mu_e < 1 + r_\beta^R$.

Lastly, we derive the three cut-offs that separate the equilibrium regimes. First, let r_R^* be the equilibrium rate of the checkable deposits in the auxiliary model where households cannot hold the CBDC. Then i_{e1}^R is the solution to

$$(1+i_e)/\mu_e - 1 = r_R^*$$

as an equation in i_e. Notice that r_R^* is also an increasing function of i_e. Nevertheless, one can show that i_{e1}^R is uniquely determined. By definition, banks just break even if $i_e = i_{e2}^R$. Therefore, it solves

$$f((1-\chi)D_e \Psi(D_e)) = (1+i_e)/\mu_e$$

as an equation in i_e. Here D_e is defined in (11) and also depends on i_e. Lastly, if $i_e = i_{e3}^R$, the loan quantity is the same as in the case without the CBDC. This means that the real loan rate is ρ^*. Because banks break even in this case, the real loan rate equals the real rate CBDC rate, which implies $i_{e3}^R = (1+\rho^*)\mu_e - 1$.

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B Multiplicity under Perfect Competition

Now we analyze a model with a perfectly competitive banking sector and illustrate the possibility of multiple equilibria if $D\Psi(D)$ is not increasing in D. To simplify the presentation, we shut down time deposits and set $\alpha_3 = 0$. The latter implies money and deposit dichotomize. For the numerical exploration later, we also introduce a marginal deposit handling cost, i.e., a bank needs to incur $c\psi d_j$ effort cost to create d_j after interest deposits. Real balances is

$$\iota = \alpha_1 \lambda(Z). \tag{15}$$

The demand for deposits d is determined by

$$\psi = \alpha_2 \beta \lambda(D) + \beta. \tag{16}$$

Notice that ψ can never go below β, and if $\psi = \beta$, D can be any value larger than y^*, and if $\psi < \beta$, $D = \infty$. To proceed, it turns out convenient to use market clearing in the deposit market. Therefore, we proceed in this way.

Suppose there is a continuum of banks with measure 1. They solve

$$\max_{\ell_j, d_j} \left\{ \left(1 + \rho - \frac{1}{\mu}\right) \ell_j - \left(1 + \psi c - \frac{\psi}{\mu}\right) d_j \right\} \tag{17}$$

$$\text{st} \quad \ell_j \leq (1 - \chi)\psi d_j. \tag{18}$$

Here we have already used the balance sheet constraint $z_j + \ell_j = \psi d_j$. If $1 + \rho > 1/\mu$, the constraint is binding. The problem becomes

$$\max_{d_j} \left\{ \left[(1 + \rho)(1 - \chi) + \frac{\chi}{\mu} - c\right] \psi d_j - d_j \right\}. \tag{19}$$

Because banks have 0 profit under perfect competition, $(1 + \rho)(1 - \chi) + \chi/\mu - c = 1/\psi$. Combine this with $1 + \rho > 1/\mu$ to obtain that the constraint is binding if $\psi < (1/\mu - c)^{-1}$. Because $\rho = \varrho(L) \equiv f'(L) - 1$, the optimization problem along with the constraint implies

$$\{1 + \varrho[(1 - \chi)\psi D]\}(1 - \chi) + \frac{\chi}{\mu} - c = 1/\psi. \tag{20}$$

This defines D as a function of ψ: $D = \Delta(\psi)$, which can be non-monotone depending on the curvature of the production function. If $\psi \geq (1/\mu - c)^{-1}$, then the constraint is not binding and $\rho = 1/\mu - 1$, $L = \varrho^{-1}(1/\mu - 1)$. If $\psi > (1/\mu - c)^{-1}$, then $D = \infty$. If

Case	α_2	ε	A	ξ	ω	$\bar{l}$
1	0.1	0	0.9	0.5	N/A	0
2	0.3	0	0.9	0.5	N/A	0
3	0.3	0.15	1.1	6	N/A	0
4	0.3	0.15	1.5	6	0.7	0.9

Table 2: Parameters for Numerical Examples

$\psi = (1/\mu - c)^{-1}$, then banks are indifferent between any amount of deposits. To satisfy the reserve requirement, $D \geq \varrho^{-1} (1/\mu - 1) (1 - \chi)^{-1} (1/\mu - c)$. The latter quantity equals $\Delta (\psi)$ if $\psi = (1/\mu - c)^{-1}$. To summarize, the supply for deposit is

$$
D = \begin{cases}
\Delta (\psi) & \psi < (1/\mu - c)^{-1} \\
\infty & \psi > (1/\mu - c)^{-1} \\
[\Delta (\psi), \infty) & \psi = (1/\mu - c)^{-1}
\end{cases}
\tag{21}
$$

Any intersection between (16) and (21) determines an equilibrium.

Proposition 5 *A steady-state monetary equilibrium exists.*

Proof. From (15), one can see that $Z > 0$ iff $\iota < \alpha_1 \lambda (0)$. If ψ is sufficiently small, (16) defines $D = \infty$ and if ψ is sufficiently large, D is sufficiently small. On the other hand, (21) implies that D is finite if ψ is low and $D = \infty$ for ψ sufficiently large. By continuity, these two curves has at least one intersection. Hence, at least one equilibrium exists. ∎

In general, the equilibrium is not unique. We next use numerical examples to illustrate this. To this end, parametrize $u(y) = [(y+0.01)^{1-\sigma} - 0.01^{1-\sigma}]/(1-\sigma)$, $f'(l) = A (l + \varepsilon)^{-\xi} \mathbf{1} \{l > \bar{l}\} + Bl^{-\omega} \mathbf{1} \{l \leq \bar{l}\}$. Here $A, \varepsilon, \bar{l} > 0$ and $\xi > 1$, $0 < \omega < 1$ are parameters to choose. Then B is chosen such that f' is continuous. One can integrate this function and impose $f(0) = 0$ to obtain f. Since f' is positive and strictly decreasing, f is strictly increasing and concave.

We consider four cases. In all cases, $\alpha_1 = 0.1$, $\beta = 0.9$, $\mu = 1.02$, $\iota = 0.02$, $c = 0.1$, $\chi = 0.1$, $\sigma = 5$. Table 2 shows other parameters. Results are shown in ψ-D space in Figure 10. The blue curve is the deposit demand curve and the red curve is the deposit supply curve. The demand curve is monotonically decreasing while the supply curve can be monotonically increasing, or non-monotone depending on the curvature of f. In case 1 and 2, the supply curves are increasing as in Figures 10(a) and 10(b). The equilibrium is unique. In case 3, the supply curve is decreasing if $\psi < (1/\mu - c)^{-1}$. This is shown in 10(c). There are three equilibria. One has $\psi = \beta$, i.e. deposit does not carry a liquidity premium. One has $\rho = 1/\mu - 1$ and $\psi = (1/\mu - c)^{-1}$. In this case, the price for deposit is sufficiently high so that
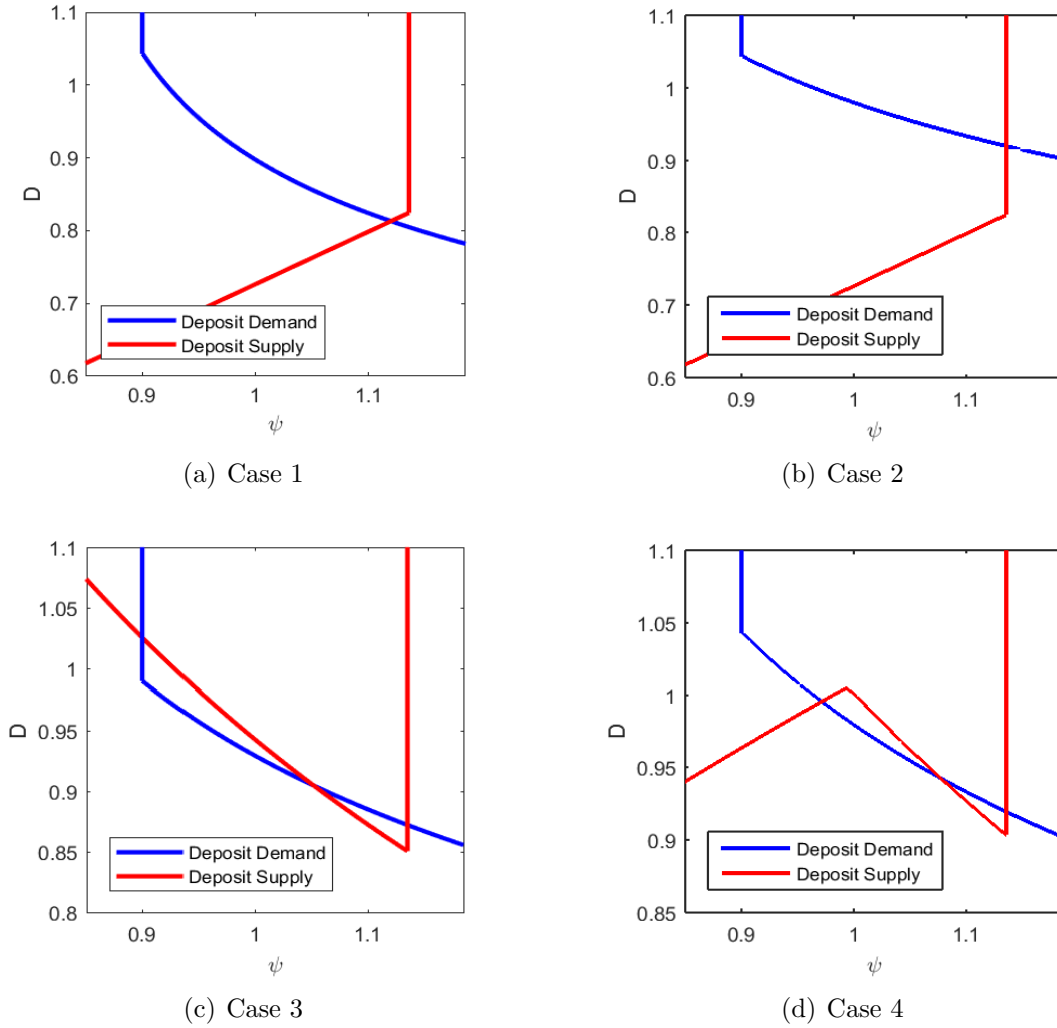
Figure 10: Equilibrium

the banks are willing to take any amount of deposits and then hold them in cash reserve. Notice at this intersection, $D > \Delta(\psi)$. Consequently, the reserve requirement constraint is not binding, i.e. banks hold voluntary reserves. There is another equilibrium where the reserve requirement constraint is binding. In case 4, the supply curve is non-monotone as shown in Figure 10(d). There are two equilibria where the reserve requirement constraint is binding. In all these examples, $D\Psi(D)$ is non-monotone, which is a necessary condition for multiplicity identified in Proposition 2. However, it is not sufficient. Multiplicity also requires a sufficiently high curvature on f.

C Imperfect Competition in Lending Market

Suppose that banks lend to entrepreneurs in a market with imperfect competition. To simplify the analysis, we also introduce a perfectly competitive interbank market to allocate resources among banks. We consider models for the imperfect competition: a Cournot model, and a search and bargaining model. In both cases, the deposit market has Cournot competition. We first derive the loan demand and the loan supply as functions of the rate in the interbank market, which is denoted by ρ^B. Then we use the market clearing condition in the interbank market to obtain the equilibrium. It turns out that this changes only the loan demand and the loan supply remains the same as in the main text. Therefore, a CBDC still pushes up the loan supply curve and can raise both deposits and loans.

C.1 Cournot Lending Market

A Bank takes the interbank market rate as given. It solves (5) with ρ replaced by ρ^B. Then the loan supply function L^s is the same as before except that it depends on ρ^B. The loan makers solve

$$\max_{\ell_j} f'\left(\ell_{-j} + \ell_j\right)\ell_j - \left(1 + \rho^B\right)\ell_j.$$

To guarantee the existence of a unique pure strategy equilibrium given ρ^B on the loan side, we require the following condition:

Assumption 2 $f''\left(L\right) + f'''\left(L\right)L \leq 0.$

Then the equilibrium loan quantity given ρ^B satisfies

$$f''\left(L\right)\frac{L}{N} + f'\left(L\right) = 1 + \rho^B.$$

This defines $L^d\left(\rho^B\right)$, which is decreasing and continuous in ρ^B. Now the equilibrium interbank market rate is determined by $L^s\left(\rho^B\right) = L^d\left(\rho^B\right)$. Then the loan rate is given by $\rho = f'\left(L^s\left(\rho^B\right)\right) - 1$.

Proposition 6 *If Assumption 2 holds, there exists at least one steady-state monetary equilibrium. If, in addition, $\Psi\left(D\right)D$ is increasing, the steady-state monetary equilibrium is unique.*

With a CBDC, the loan supply remains the same as in the main text but with ρ^B replacing ρ. The loan demand is the same as the one derived above. Following the same analysis in the main text, one can conclude that a CBDC can promote bank intermediation.

C.2 Search for Loans

Now suppose that each bank has a continuum of loan officers with measure N_l. They have access to a competitive interbank market and randomly search and match with firms. The matching probability is $\alpha(\lambda)$, where $\lambda = 1/N_l N_b$. Upon a meeting, the loan officer bargains with the firm on the terms of loans given the interbank market rate ρ^B. The surplus is split with Kalai's bargaining solution, where the firm has a bargaining power η:

$$\max_{\ell,p} f(\ell) - p$$
$$\text{st} \quad f(\ell) - p = \eta \left[f(\ell) - \left(1 + \rho^B\right) \ell \right].$$

The solution is $f'(\ell) = \ell + \rho^B$ and $p = (1 - \eta) f(\ell) + \left(1 + \rho^B\right) \ell$. This implies the loan rate,

$$\rho^E = \eta \rho^B + (1 - \eta) \left[\frac{f(\ell)}{\ell} - 1 \right],$$

which is a weighted sum of the interbank market rate ρ^B and the average investment return $f(\ell)/\ell - 1$. The loan demand curve is

$$L^d\left(\rho^B\right) = \alpha(\lambda) N_l N_b f'^{-1} \left(1 + \rho^B\right).$$

Then the equilibrium ρ^B is determined in the same fashion as in the competitive loan market case. Consequently, we have the following proposition:

Proposition 7 *There exists at least one steady-state monetary equilibrium. If, in addition, $\Psi(D) D$ is increasing, then the steady-state monetary equilibrium is unique.*

Notice that in this case, the total supply of loans is efficient given the interbank market rate ρ^B. However, banks get a positive surplus from lending. We can then calculate the spread between the interbank rate and lending rate as $\rho^E - \rho^B = (1 - \eta) \left[\frac{f(\ell)}{\ell} - 1 - \rho^B \right]$. Again, a CBDC can promote bank intermediation in the same way as in the main text.

D Price Competition in the Deposit Market

Consider an alternative deposit market structure, where banks set the real price or interest on deposits. They have market power due to information frictions following Burdett and Judd (1983) and Head et. al (2012). There are a continuum of banks. Each of them quote prices for its checkable deposits and time deposits. Due to information frictions, a buyer does not see all the price postings. Instead, he/she sees one quote with probability b_1 and two quotes

with probability b_2. For simplicity, assume that $b_1 + b_2 = 1$. If the buyer sees two quotes, he/she chooses to stay with the bank that quotes a lower price. After choosing the bank, the buyer works and makes portfolio choices. There can be heterogeneity in the portfolio choices because buyers may face different interest rates on their deposits. Nevertheless, the inverse demand function for a buyer remains to be Ψ. It is convenient to work with $\mathbf{D}(\psi) = \Psi^{-1}(\psi)$.

Banks engage in perfect competition in the loan market. Given the loan rate, they choose ψ and ψ_b to maximize the expected profit. Same as before, banks offer time deposit only if $\rho \geq 1/\beta - 1$. If $\rho = 1/\beta - 1$, they are indifferent between any amount of time deposits and $\psi_b = 1/\beta$. Therefore, we can focus only on the choice of ψ.

Following Head et. al (2012), there is a continuum of ψ quoted in the equilibrium. The all lead to the same expected profit. Banks trade off the profit from a customer and the probability of getting a customer. Denote the distribution of ψ as F. One can show that F is non-atomic and has an interval support. The highest price in the equilibrium solves

$$\max_{z, \ell, \psi} b_1 \left\{ \left[(1 + \rho) \ell + \frac{z}{\mu} \right] \psi \mathbf{D}(\psi) - \mathbf{D}(\psi) \right\}$$
$$\text{st} \quad z + \ell = \psi \mathbf{D}(\psi),$$
$$z \geq \chi \psi \mathbf{D}(\psi).$$

A bank with the highest price gets a customer only if the customer sees only one quote. This happens with probability b_1. Conditional on having a customer, the problem is the same as before with $D_{-j} = 0$, i.e., the bank is a local monopoly. This problem can be rewritten as

$$\max_{\psi} b_1 (\xi \psi - 1) \mathbf{D}(\psi).$$

where $\xi = \max \{ (1 + \rho)(1 - \chi) + \chi/\mu, 1/\mu \}$.

Assumption 3 $\xi \mathbf{D}(\psi) + (\xi \psi - 1) \mathbf{D}'(\psi) = 0$ *has a unique solution for every* ρ.

Under Assumption 3 , $b_1 (\xi \psi - 1) \mathbf{D}(\psi)$ has a unique maximizer $\bar{\psi}(\rho)$ which satisfies that

$$\xi \mathbf{D}(\bar{\psi}(\rho)) + \left[\xi \bar{\psi}(\rho) - 1 \right] \mathbf{D}'(\bar{\psi}(\rho)) = 0.$$

The distribution of ψ satisfies the equal profit condition

$$b_1 \left[\xi \bar{\psi}(\rho) - 1 \right] \mathbf{D}(\bar{\psi}(\rho)) = \{ b_1 + 2b_2 [1 - F(\psi; \rho)] \} (\xi \psi - 1) \mathbf{D}(\psi).$$

The left hand side is the profit from quoting the highest price. The right hand side is the profit from quoting a lower price ψ. A bank with price ψ gets the customer if either the customer has only its quote or his/her other quote has a higher price. This equal profit condition gives a closed-form solution of F given $\bar{\psi}(\rho)$

$$F(\psi; \rho) = 1 - \frac{b_1}{2b_2}\left\{\frac{\left[\xi\bar{\psi}(\rho) - 1\right]\mathbf{D}\left(\bar{\psi}(\rho)\right)}{[\xi\psi - 1]\mathbf{D}(\psi)} - 1\right\}.$$

Denote the lowest price in the support of F as $\underline{\psi}(\rho)$. It solves $F(\psi; \rho) = 0$. Noitce that if $\rho \leq 1/\beta$, then $\underline{\psi}(\rho) > \beta$. The accepted quotes have the price distribution

$$G(\psi; \rho) = b_1 F(\psi; \rho) + b_2\left\{1 - [1 - F(\psi; \rho)]^2\right\}.$$

Again, banks hold only cash if $1 + \rho < 1/\mu$, and lend up to the reserve requirment if $1 + \rho > 1/\mu$ and are indifferent if $1 + \rho = 1/\mu$. The aggregate loan supply is

$$L^s(\rho) = \begin{cases} 0 & \text{if } \rho < \frac{1}{\mu} - 1 \\ \left[0, (1 - \chi)\int\psi\mathbf{D}(\psi)\,dG(\psi; \rho)\right] & \text{if } \rho = \frac{1}{\mu} - 1 \\ (1 - \chi)\int\psi\mathbf{D}(\psi)\,dG(\psi; \rho) & \text{if } 1/\beta - 1 > \rho > \frac{1}{\mu} - 1 \\ \left[(1 - \chi)\int\psi\mathbf{D}(\psi)\,dG(\psi; \rho), \infty\right] & \text{if } \rho = 1/\beta - 1 \end{cases}.$$

It takes the same form as the case with Cournot competition.

Proposition 8 *There exists at least one steady-state monetary equilibrium if Assumption 3 holds. If, in addition, $D\Psi(D)$ is increasing and $\bar{\psi}(\rho)$ is decreasing, the equilibrium is unique.*

Proof. Notice that $L^s(\rho)$ is continuous and equals 0 if ρ is small and can take any value between $(1 - \chi)\int\psi\mathbf{D}(\psi)\,dG(\psi; \rho)$ and ∞ if $\rho = 1/\beta - 1$. The loan demand curve is the same as in the main text. Therefore, $L^s(\rho) < L^d(\rho)$ if ρ is sufficiently small and $\max L^s(\rho) > L^d(\rho)$ if $\rho = 1/\beta - 1$. By the intermediate value theorem, there is at least one solution to $L^s(\rho) - L^d(\rho) = 0$.

Notice that for all $\psi < \bar{\psi}(\rho)$,

$$\frac{\partial}{\partial\rho}F(\psi; \rho) \simeq \frac{\partial\xi}{\partial\rho}\mathbf{D}\left(\bar{\psi}(\rho)\right)\mathbf{D}(\psi)\left[\bar{\psi}(\rho) - \psi\right] \geq 0.$$

In addition, $\bar{\psi}(\rho)$ decreases with ρ. As a result, $F(\psi; \rho)$ weakly increases for all ψ as ρ increases, so does $G(\psi; \rho)$. Moreover, if $D\Psi(D)$ is increasing in D, $\psi\mathbf{D}(\psi)$ is decreasing

in ψ because Ψ is decreasing. This implies that $\int \psi \mathbf{D}(\psi) \, dG(\psi; \rho)$ is non-decreasing in ρ. Therefore, $L^s(\rho)$ is non-decreasing and $L^s(\rho) - L^d(\rho)$ is increasing, which ensures uniqueness. ∎

Now introduce the baseline CBDC with $1/\mu < \frac{\mu_e}{1+i_e} < 1/\beta$ as in the main text. Households always have access to the CBDC. They compare the CBDC rate with quotes from banks and pick the one that gives a higher return. Then, the CBDC puts a cap on ψ, i.e. $\psi \leq \frac{\mu_e}{1+i_e}$ and the highest price that banks charge is $\min \left\{ \frac{\mu_e}{1+i_e}, \bar{\psi}(\rho) \right\}$. Again, define $\underline{\rho}$ as the solution to

$$(1-\chi)(1+\rho) + \chi/\mu = \frac{1+i_e}{\mu_e}.$$

Let $\bar{\rho}$ satisfy $\bar{\psi}(\bar{\rho}) = \frac{\mu_e}{1+i_e}$. Let $\tilde{F}(\psi; \rho)$ be the distribution of ψ with the CBDC and

$$\tilde{G}(\psi; \rho) = b_1 \tilde{F}(\psi; \rho) + b_2 \left\{ 1 - \left[1 - \tilde{F}(\psi; \rho) \right]^2 \right\}.$$

If $\rho < \underline{\rho}$, banks do not operate. If $\rho \geq \bar{\rho}$, $\bar{\psi}(\rho) \leq \frac{\mu_e}{1+i_e}$ and the CBDC does not change the distribution of ψ, i.e., $\tilde{F}(\psi; \rho) = F(\psi; \rho)$. If $\underline{\rho} < \rho < \bar{\rho}$, the distribution of ψ is

$$\tilde{F}(\psi; \rho) = 1 - \frac{b_1}{2b_2} \left[\frac{\left(\xi \frac{\mu_e}{1+i_e} - 1 \right) \mathbf{D}\left(\frac{\mu_e}{1+i_e} \right)}{(\xi\psi - 1) \mathbf{D}(\psi)} - 1 \right]. \tag{22}$$

Obviously, $\tilde{F}(\psi; \rho) \geq F(\psi; \rho)$ for all ψ in this case. If $\rho = \underline{\rho}$, $\psi = \mu_e/(1+i_e)$ is degenerate. Then banks are indifferent among any deposit and loan quantities. On the other hand, households are indifferent between the CBDC and checkable deposits. As a result, the deposit quantity can be anything between 0 and $\frac{\mu_e}{1+i_e} \Psi\left(\frac{\mu_e}{1+i_e} \right)$. The aggregate loan supply is

$$\tilde{L}^s(\rho) = \begin{cases} 0 & \text{if } \rho < \underline{\rho} \\ \left[0, (1-\chi) \frac{\mu_e}{1+i_e} \Psi\left(\frac{\mu_e}{1+i_e} \right) \right] & \text{if } \rho = \underline{\rho} \\ (1-\chi) \int \psi \mathbf{D}(\psi) \, d\tilde{G}(\psi; \rho) & \text{if } \bar{\rho} > \rho > \underline{\rho} \\ (1-\chi) \int \psi \mathbf{D}(\psi) \, dG(\psi; \rho) & \text{if } \frac{1}{\beta} - 1 > \rho \geq \bar{\rho} \\ \left[(1-\chi) \int \psi \mathbf{D}(\psi) \, dG(\psi; \rho), \infty \right] & \text{if } \rho = 1/\beta - 1 \end{cases}.$$

Figure 11 shows the loan demand and loan supply curves if $D\Psi(D)$ is increasing. The black curve is the aggregate loan supply curve without the CBDC and the red curve is the curve with the CBDC. Similar to the Cournot model, the red curve is above the black curve if $\rho \in (\underline{\rho}, \bar{\rho})$ and overlaps with the black curve if $\rho > \bar{\rho}$. Different from the Cournot model, it
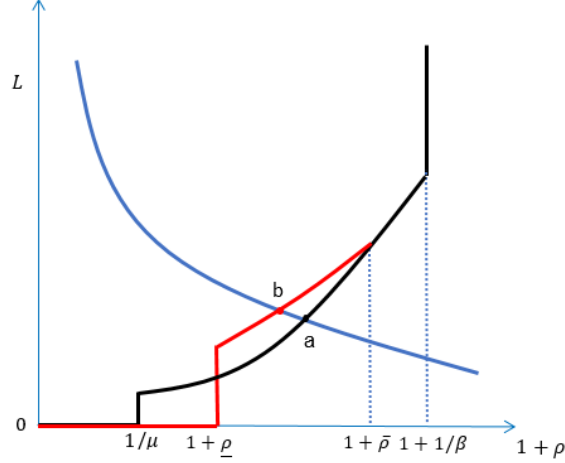
Figure 11: Regime 2 under the Burdett-Judd Deposit Market

is increasing on $\left(\underline{\rho}, \bar{\rho}\right)$. Using (22), one can show that $\tilde{F}\left(\psi; \rho\right)$ is increasing with ρ on $\left(\underline{\rho}, \bar{\rho}\right)$. As a result, $\hat{L}^s\left(\rho\right)$ is strictly increasing with ρ if $D\Psi\left(D\right)$ is increasing. The blue curve is the loan demand curve. Its intersections with the loan supply curves correspond to equilibria with and without the CBDC. In this figure, we plot the case where i_e is intermediate. The equilibrium with the CBDC (point b) has a higher loan quantity and a lower loan rate than the equilibrium without the CBDC (point a). Because the equilibrium is between $\underline{\rho}$ and $\bar{\rho}$, households do not use the CBDC in equilibrium despite that it has a positive effect on bank intermediation. Same as in the Cournot model, a higher i_e increases both $\underline{\rho}$ and $\bar{\rho}$. As $\left(1 + i_e\right)/\mu_e$ increases from $1/\mu$, the economy goes through the same four regimes as described in Figure 4. The CBDC first increases bank intermediation and then decreases bank intermediation.

E   Calibration Method and Data

In the calibration, we use Kalai bargaining as the DM trading mechanism. It is more flexible and allows sellers in the DM to have positive mark-ups. The bargaining power to the buyer is $\theta \in [0, 1]$. The solution maximizes the buyer's surplus given that he or she gets θ fraction of the total surplus and the liquidity constraint, i.e., if the buyer has a real balance $\mathcal{L}$, the Kalai solution solves

$$\max_{y,p} \left[u\left(y\right) - p\right] \text{ s.t. } u\left(y\right) - p = \theta\left[u(y) - y\right] \text{ and } p \leq \mathcal{L}.$$

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All the analysis in the main text stays unchanged except that

$$\lambda(\mathcal{L}) = \max\left\{\frac{u'[Y(\mathcal{L})]}{(1-\theta)u'[Y(\mathcal{L})]+\theta} - 1, 0\right\},$$

where $Y(\mathcal{L})$ satisfies $(1-\theta)u[Y(\mathcal{L})] + \theta Y(\mathcal{L}) = \mathcal{L}$. If $\theta = 1$, Kalai bargaining reduces to the buyer take-it-or-leave it offer studied in the main text.

Data

From FRED, we obtain the time series for inflation, 3 month t-bill rates, prime rates, GDP and total commercial loans. The SCPC data contain the number of transactions by type. Table 9 in Greene and Stavins (2017) contains these numbers for 2015-2017. DCPC asks consumers to record whether they think a transaction accepts cash or cards. Page 13 and 14 in Premo (2018) contain summary statistics of answers to these questions.

For calibration, we need times series of interest rates on transaction deposits and loan, and information on the operation costs of banks. We obtain them from call reports data from 1987-2019. This data contain quarterly information on balance sheet and income statement of banks in the US. We obtain this data from WRDS by using the SARS code by Drechsler et al. (2017). To obtain the rates on transaction deposits, we first divide interest expenses on transaction accounts (item code: RAID 4508) by total transaction deposits (RCON2215) to obtain the rates for each bank in a given quarter. Then we take the average across all banks weighted by their transaction deposits to obtain a quarterly industry average. Lastly, we aggregate to the annual level.

To obtain the loan rates, we first divide the interest income from loans (RIAD4010) by the loan quantity (RCON3360) to obtain bank-level loan rates. These rates are very heterogeneous across banks. This could be because loans of different banks have different risk. Since we do not model risky investment, we focus only on the safe loans. We define our loan rate to be the first percentile of the loan rate distribution in the data. This leads to rates comparable to the FRED on loans of minimal risk, which is reported between 1998 and 2008. In this period, our average loan rate is about 4.4%, while the FRED data have an average rate of 4.7%. This gives a 3.69% loan rate. We have also done a calibration with the average loan rate 5.19%. The results are similar but the positive effect of the CBDC is larger because the higher loan rate implies a higher market power in the banking sector.

Lastly, we compute the operational cost per dollar of asset between 2014 and 2019 to calibrate c. Unfortunately, assets data are missing for many banks during this time frame. But we

observe total deposits (RCON2200+RCFN2200) for this period. We also observe both assets and deposits between 1987 and 2010. In this period, assets is about 1.505 times the total deposits. We then assume that the this ratio is stable over time. Therefore, we can divide the operational cost per dollar of deposits by 1.505 to obtain operational cost per dollar of asset. To this end, We first calculate the average operational cost for each bank by subtracting expenses on premises or rent (RIAD4217) from the non-interest expenses (RIAD4903). Then take an average across banks weighted by total deposits to get the industry average. Then aggregate to annual level and set c to be the time average divided by 1.505.

Computation

One straightforward way to calibrate the model is to solve the equilibrium given each parameter value and choose one that best fits the money demand curve, the deposit rates and the spread. This method, however, can be computationally cumbersome because one needs to solve the model for each data point used for the money demand and then optimize over a six-dimensional parameter. One key insight is that the money demand can be solved independent of the banking sector. This leads to the following algorithm that greatly simplifies the calibration.

1. Match the money demand between 1987 and 2008 to obtain $B, \sigma, \theta, \Omega$.

   (a) Fix the value of Ω and θ. Fit the money demand curve by choosing B, σ. More specifically, for each interest rate, calculate the steady state equilibrium using the nominal interest rate and the deposit rate for each year. Then choose (B, σ) to minimize the distance between the model predicted M1 to GDP ratio and the data. The M1 to GDP ratio in the model can be calculated by $[Z + \Psi(D)D]/\mathbf{Y}$, where

   $$\mathbf{Y} = \sum_{j=1}^{3} \alpha_j P(y_j) + 2B + A\left[(1-\chi) D\Psi(D)\right]^\eta - D + (1-\chi) D\Psi(D)$$

   is the output. It is the sum of the consumption of households in DM and CM, the consumption of bankers and entrepreneurs, and the investment. The DM consumption is measured by the amount of payment. The first-order condition of the entrepreneurs implies $A\eta\left[(1-\chi) D\Psi(D)\right]^{\eta-1} = 1 + \rho$. Therefore,

   $$\mathbf{Y} = \sum_{j=1}^{3} \alpha_j P(y_j) + 2B + (1+\rho)(1-\chi) D\Psi(D)/\eta - D + (1-\chi) D\Psi(D).$$

This formulation does not involve A. We plug in the time series for ρ. This insight allows us to calibrate B, σ independent of the bank's problem.

(b) Calculate the markup. If it is less than 20%, then decrease θ, otherwise increase θ. Repeat 1-2 until the markup in the model matches 20%.

(c) Calculate the model fit at different values of Ω. And find the value that gives the best fit.

2. Match banking data from 2014 to 2018 to obtain N, A

(a) Set N such that the solution of the Cournot competition leads to a spread of 3.3% between loans and transaction account.

(b) Set A to match a 0.3049% interest rate on transaction accounts.

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